Low-complexity Channel Polarized Multilevel Coding for Modulation-format-independent Forward Error Correction

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Abstract *We propose a modulation-format-independent binary coding scheme that applies multilevel coding to channel polarized signals with different capacities. The simulation results show that the proposed method can reduce complexity with slight net coding gain degradation by applying SD-FEC only to non-reliable bits compared to BICM.*

Introduction

In order to meet the rapidly growing data traffic demands of optical networks, the application area of digital coherent optical communication is expanding to metro and data center interconnect (DCI) in addition to long-haul networks[1]. In coherent optical transmission, the modulation order and forward error correction (FEC) schemes are designed according to the demands of transmission distance and data rate. For example, in long-haul networks, strong SD-FEC and QPSK are used to extend the transmission distance. In contrast, for metro and data center interconnects, the application of higher order modulations and low-complexity FEC is required to optimize a spectral efficiency and power consumption[2].

Bit interleaved coded modulation[2] (BICM) shown in Fig. 1(a) is a practical method to design FEC under various modulation formats. BICM makes a channel capacity of symbol bit-levels uniform by using a bit-interleaver and can design a soft-decision FEC (SD-FEC) utilizing powerful families of binary codes such as LDPC, Polar, and Turbo codes without depending on the modulation format[2]. However, adopting SD-FEC, which has a high decoding complexity, on the entire FEC frame in BICM, the power consumption of FEC increases. In fact, BICM with SD-FEC is one of the major contributors to power consumption in coherent DSPs[3],[4].

The multilevel coding (MLC) scheme has recently been attracting attention in the optical communication[5]–[8]. Two-level MLC[9] (TL-MLC) configuration is shown in Fig. 1(b) as an example. TL-MLC uses multistage decoding (MSD) and non-Gray labeling to make the channel capacity of the symbol bit-level non-uniform and assigns strong SD-FEC to only the non-reliable least significant bit (LSB) to reduce complexity. The outer-FEC code using hard decision FEC (HD-FEC) corrects both the error for most significant bits (MSBs) and the residual bit errors for LSB. However, in TL-MLC with low modulation orders, the complexity reduction is less than higher order modulation because of the large ratio of LSB to the entire FEC frames[5]–[8].

In this work, we propose a channel polarized MLC (CP-MLC) that reduces the complexity without depending on the modulation order by using channel polarization[9]. CP-MLC is composed of a binary code framework, which means it can construct a low-complexity FEC independent of modulation format by combining it with BICM. We constructed a FEC using CP-MLC on QPSK and 16QAM. Our numerical simulations on the AWGN channel along with a complexity measurement evaluation shows that CP-MLC can reduce the complexity without depending on the modulation order with less net coding gain (NCG) degradation.

Channel Polarized MLC

CP-MLC uses channel polarization to transform an original channel into multiple sub-channels with different capacities and only assigns SD-FEC to a non-reliable sub-channel to reduce the complexity.

Figure 1 (c) shows the procedure of the proposed CP-MLC at the transmitter side. First,

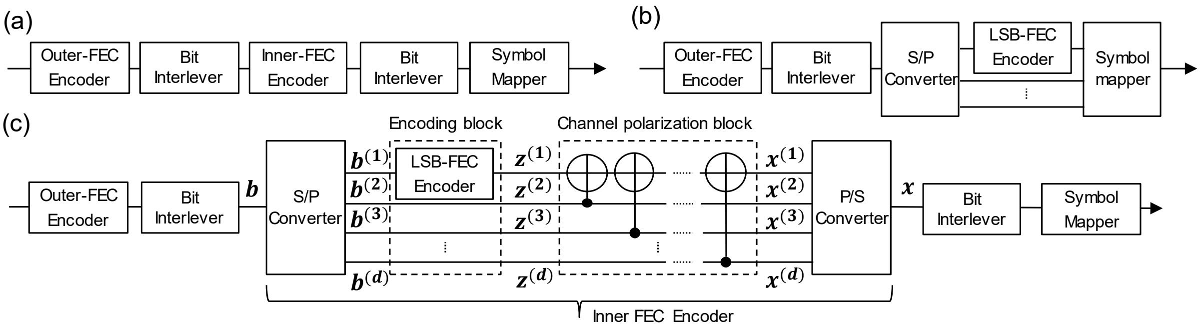

Fig. 1: Block diagram of (a) BICM, (b) TL-MLC, and (c) proposed CP-MLC.

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the outer-FEC encoder converts information bits into the outer-FEC codeword $\boldsymbol{b}$. Next, in the inner-FEC encoder, the serial/parallel (S/P) converter divides $\boldsymbol{b}$ into d blocks, and we obtain bit sequences $\boldsymbol{b}^{(1)}, \boldsymbol{b}^{(2)}, \dots, \boldsymbol{b}^{(d)}$ with the length of $k', n', n', \dots, n'$ bits, respectively. With abuse of notation, we call the lane of $\boldsymbol{b}^{(1)}$ to LSB and the others to MSBs. In the encoding block, the LSB-FEC encoder converts $\boldsymbol{b}^{(1)}$ to the codeword $\boldsymbol{z}^{(1)}$ with the length of n' bits by using SD-FEC. MSBs are not encoded ($\boldsymbol{z}^{(i)} = \boldsymbol{b}^{(i)}$, where $i \neq 1$). Next, the CP block, which is reversible transformation, calculates as $\boldsymbol{x}^{(1)} = \boldsymbol{z}^{(1)} \oplus \boldsymbol{z}^{(2)} \oplus \cdots \oplus \boldsymbol{z}^{(d)}$ in LSB, where $\oplus$ is the exclusive OR for each element of the vector. MSBs, $\boldsymbol{x}^{(i)}$ are output as $\boldsymbol{z}^{(i)}$, where $i \neq 1$. After that, the P/S converter combines each $\boldsymbol{x}^{(i)}$ to make a serial bit sequence $\boldsymbol{x}$. Since CP-MLC output a bit sequence $\boldsymbol{x}$, it is a binary FEC code. Thus, CP-MLC can be designed low-complexity FEC independently of the modulation format by combining with coded modulation such as BICM.

In CP-MLC, a strong SD-FEC only use on the LSB and the outer-FEC code corrects both MSBs and residual errors for LSB. Therefore, LSB and MSBs need to be non-reliable and reliable bits, respectively. Here, we describe that the CP-block respectively converts LSB and MSBs into non-reliable and reliable bits; the channel capacity between the LSB and MSBs becomes non-uniform. For simplicity, we consider a symmetric binary-input memoryless channel where binary-AWGN divides d channels $W: X \to Y$. We denote $\overline{\boldsymbol{X}} = (X^{(1)}, X^{(2)}, \dots, X^{(d)})$, where $X^{(1)}$ is a random variable corresponding to bit of $\boldsymbol{x}^{(1)}$, and use the same notation for $\overline{\boldsymbol{Y}}$ and $\overline{\boldsymbol{Z}}$. Consider the sub-channel of LSB $W_1: Z^{(1)} \to \overline{\boldsymbol{Y}}$ and the sub-channels of MSBs $W_i: Z^{(i)} \to \overline{\boldsymbol{Y}}, Z^{(1)}$. We denote the capacity of W, W_1 and W_i by $I(W)$, $I(W_1) = I(\overline{\boldsymbol{Y}}; Z^{(1)})$, and $I(W_i) = I(\overline{\boldsymbol{Y}}; Z^{(1)} | Z^{(i)})$ respectively. Then, we have

$$
\begin{aligned}
d \times I(W) &= I(\overline{\boldsymbol{Y}}; \overline{\boldsymbol{Z}}) \\
&= I(\overline{\boldsymbol{Y}}; Z^{(1)}) + \sum_{i=2}^{d} I(\overline{\boldsymbol{Y}} | Z^{(1)}; Z^{(i)}) \\
&= I(\overline{\boldsymbol{Y}}; Z^{(1)}) + \sum_{i=2}^{d} I(\overline{\boldsymbol{Y}}, Z^{(1)} | Z^{(i)}) \\
&= I(W_1) + (d-1) I(W_2). \qquad (1)
\end{aligned}
$$

Here, the first equation uses mutual information conserved because CP-block is reversible transformation $\overline{\boldsymbol{Z}} \to \overline{\boldsymbol{X}}$. The second equation uses the chain law of mutual information and the fact that $W_2, W_3, \cdots, W_d$ are independent and identically distributed. The third equation uses $I(Z^{(1)}; Z^{(i)}) + I(\overline{\boldsymbol{Y}} | Z^{(1)}; Z^{(i)}) = I(\overline{\boldsymbol{Y}}, Z^{(1)}; Z^{(i)})$ and $I(Z^{(1)}; Z^{(i)}) = 0$. Also,

$$
\begin{aligned}
I(W_2) = \cdots = I(W_d) &= I(\overline{\boldsymbol{Y}}, Z^{(1)}; Z^{(2)}) \\
&\geq I(Y^{(2)}; Z^{(2)}) \\
&= I(Y^{(2)}; X^{(2)}) = I(W) \qquad (2)
\end{aligned}
$$

holds. In $I(Y^{(2)}; Z^{(2)}) = I(Y^{(2)}; X^{(2)})$, we use $X^{(2)} = Z^{(2)}$. Therefore, we have the following from Eqs. (1) and (2):

$$d \times I(W) - (d-1) I(W_2) \geq I(W) \geq I(W_1). \qquad (3)$$

Thus, from Eqs. (2) and (3),

$$I(W_2) = \cdots = I(W_d) \geq I(W) \geq I(W_1). \qquad (4)$$

holds. Eqs. (3) and (4) show that the channel capacity between the LSB and MSBs becomes non-uniform. That is, the FECs for CP-MLC can design the LSB-FEC and the outer-FEC code in a similar manner as those for TL-MLC.

Next, we explain the procedure of decoding for CP-MLC at the receiver side shown in Fig. 2.

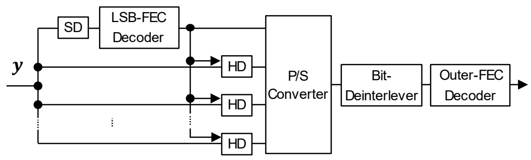

Fig. 2: Block diagram of decoder for CP-MLC.

We denote each bit of $\boldsymbol{z}^{(1)}$ by $z_j^{(1)}$. The log-likelihood ratio (LLR) of $p(\boldsymbol{y} | z_j^{(1)})$ is obtained from both the received word $\boldsymbol{y}$ and the channel information $p(\boldsymbol{y}|\boldsymbol{x})$ in soft-decision (SD) block, Then, LSB-FEC decoder estimates $\boldsymbol{z}^{(1)}$ from the LLR. In MSBs, $z_j^{(i)}$ is estimated by hard-decision (HD) for LLR of conditional probabilities $p(\boldsymbol{y}, z_j^{(1)} | z_j^{(i)})$ calculated by both received word $\boldsymbol{y}$, $p(\boldsymbol{y}|\boldsymbol{x})$ and $z_j^{(1)}$ in the HD block. Then, the outer-FEC decoder corrects both the error for MSBs and the residual bit errors for LSB after P/S convertor and bit-deinterleaver.

Next, we describe the measure used for complexity evaluation of CP-MLC. Accurate FEC-complexity evaluation is difficult since it depends on implementation of hardware circuit. In [7] and [11]–[13], the complexity of an LDPC codes is evaluated by using the measure η. In this work, we evaluate the measure η with the division number d which is the number of output bit sequence from S/P converter (see Fig. 1(c)). Consequently, the complexity of CP-MLC is defined by

$$\eta_{CP-MLC} \coloneqq \frac{(1 - R_{LSB-FEC})(\bar{d}_c - v) I}{d - 1 + R_{LSB-FEC}} + P, \qquad (5)$$

where $R_{LSB-FEC}$ is the coding rate of the LSB-FEC (LDPC code), $\bar{d}_c$ is the average check node (CN) degree of the LDPC code, v is the average of the variable nodes of degree 1 connected to CN, I is the maximum number of iterations of the sum-product algorithm (SPA), and P is the complexity of HD-FEC. Note that P is equal to 0 in this study since the complexity of HD-FEC is sufficiently lower than the LDPC code[15].

Simulation Setup and Results

Table 1 shows the simulation parameters to evaluate NCG and complexity for BICM, TL-MLC and the proposed CP-MLC with the frame length of 30240 and the total overhead (OH) of around 14.5%. For CP-MLC, the OHs of inner- and outer-FEC codes were designed to optimize a NCG because the ratio between non-reliable LSB and reliable MSBs changed with division number. SD-FEC was the regular LDPC code with SPA, and the maximum number of iterations was set to 16. We assumed that the outer-FEC code, which corrected both the error for MSBs and the residual bit errors for LSB, was connected to the inner-FEC code. Here, the threshold of the pre-outer-FEC bit error ratio (BER) for the post-outer-FEC BER to achieve 10^{-15} included a margin with respect to the theoretical threshold of outer-FEC codes[14]–[15] since it depends on the performance deterioration by the finite length bit-interleaver between the inner- and outer-FEC codes[7].

Figure 3 shows the decoding performance of the inner-FEC code in QPSK and 16QAM under the AWGN channel. The error floor occurred with increasing division number d as shown in Fig. 3. Since the inner-FEC code corrects only the LSB, the post-inner-FEC BER is dominated by the BER of MSBs. Therefore, increasing division number d required the outer-FEC code with a high OH because of increasing the proportion of MSBs in the entire FEC frame.

Figure 4 shows that the NCG versus η for BICM, TL-MLC, and CP-MLC (division number: d=2, 4, and 6). In order to calculate each NCG shown in Tab. 1 and Fig. 4, the required E_s/N_0 for the post-outer-FEC BER to achieve 10^{-15} was obtained by post-inner-FEC BER and the threshold of pre-outer-FEC BER. The complexities of CP-MLC for division number of 2, 4, and 6 respectively were 51%, 76%, and 84% lower than that of BICM in both QPSK and 16QAM. The deterioration of NCG for QPSK were 0.29, 0.45, and 0.49 dB, and for 16QAM were 0.05, 0.11, and 0.17 dB when division number were 2, 4, and 6. We also evaluated NCG and complexity for the TL-MLC. The TL-MLC and the CP-MLC with the division number of 2 had the same NCG and complexity as shown in Fig. 4. From the simulation results in Fig. 4, CP-MLC can reduce the complexity without depending on the modulation order with less NCG degradation. Note that the complexity reduction scheme by CP-MLC can combine with other schemes such as changing a maximum iteration number[16] and decoding algorithm[17].

Conclusions

In this paper, we proposed a CP-MLC to reduce the decoding complexity by using channel polarization that transforms into sub-channels having non-uniform capacities and only assigns strong SD-FEC to a non-reliable sub-channel. CP-MLC can reduce the complexity without depending on modulation format because it belongs to a binary codes framework. The complexities of CP-MLC for division number of 2, 4, and 6 respectively were 51%, 76%, and 84% lower than that of BICM. The simulation results of CP-MLC for QPSK and 16QAM show slight net coding gain degradation compared to BICM.

Tab. 1: Simulation parameters and results (NCG and complexity).

Scheme	SD-FEC ratio	FEC-OH	SD-FEC code	Inner-FEC OH	LSB-FEC OH	Outer-FEC code	Pre-outer-FEC BER threshold	OH Outer-FEC OH	NCG [dB] QPSK	NCG [dB] 16QAM	Compl-exity η
BICM	1	14.87%	(30240,25202)LDPC	14.28%	14.28%	BCH	1×10^{-6}	0.52%	11.30	11.77	54.82
TL-MLC	1/2	14.54%	(15120,12098)LDPC	11.10%	24.98%	zipper[14]	1×10^{-3}	3.09%	-	11.72	26.65
CP-MLC (d=2)	1/2	14.54%	(15120,12098)LDPC	11.10%	24.98%	zipper[14]	1×10^{-3}	3.09%	11.01	11.72	26.65
CP-MLC (d=4)	1/4	14.82%	(7560,5042)LDPC	9.09%	49.9%	zipper[14]	3×10^{-3}	5.26%	10.85	11.66	13.08
CP-MLC (d=6)	1/6	14.87%	(5040,2882)LDPC	7.69%	74.9%	Staircase[15]	5×10^{-3}	6.67%	10.81	11.60	8.61

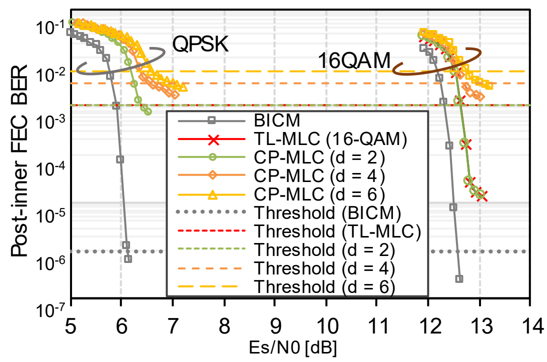

Fig. 3: Post-inner-FEC BER versus E_s/N_0 for QPSK and 16QAM. Dashed lines are the threshold of pre-outer-FEC BER.

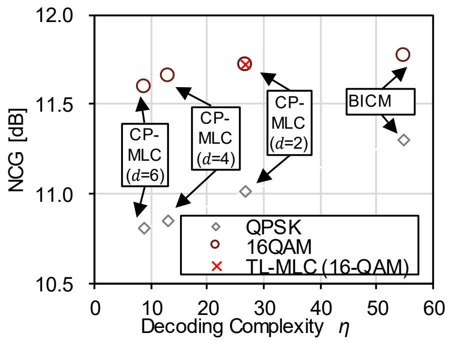

Fig. 4: NCG versus decoding complexity

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