Extending White Gaussian Noise Capacity Estimation Method with Probabilistic Constellation Shaping

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Abstract *White Gaussian noise is used as a test signal in order to estimate transmission capacity. Based on the received noise signal, we design a probabilistically shaped constellation by mapping the received noise to constellation points and show good agreement with theoretical capacity.*

Introduction

With optical transmission systems moving towards higher order quadrature amplitude modulation (QAM) formats and the introduction of probabilistic constellation shaping (PCS)[1] and geometric shaping[2], signals coming from an optical transmitter are converging to a Gaussian distribution. It is known that for an additive white Gaussian noise (AWGN) channel a white Gaussian noise (WGN) signal achieves the maximum capacity. Traditionally, WGN has been used as loading channels in transmission experiments to emulate channel interference[3]. However, observing that WGN maximizes the capacity, in[4] WGN was used as a test signal in order to estimate transmission capacity.

For capacity estimation using WGN, a noise band is generated and split into a reference and test path. Using digital signal processing (DSP), the error between the reference and test path is determined and used to generate an error signal which can be applied to an arbitrary signal, as long as the bandwidth of that signal is smaller than the receiver bandwidth. Using WGN as test signal has inherent advantages, as it has an infinite pattern length and it is baudrate agnostic[4]. Also, since the signal originates from spontaneous emission rather than a conventional transmitter, test signal quality is not degraded by transmitter component impairments such as laser phase noise, digital-to-analog converter (DAC) effective number of bits (ENOB), RF-amplifier noise and the IQ-modulator bandwidth.

It is paramount to choose an optimum constellation for the specific signal-to-noise ratio (SNR) tested in order to approach the Shannon capacity. In[4], multi-ring constellations[5] were used to approach an optimal constellation, while in this paper we design an optimal constellation by using the received WGN to create a probabilistically shaped (PS) constellation. We compare performance of the WGN capacity estimation method for PS constellations and uniform QAM constellations both in simulation and experimentally. Demonstrating the use of the designed PS constellations with the WGN capacity estimation method, we show 1.4 dB gain over 1024-QAM and indeed approach Shannon.

Capacity Estimation Using White Gaussian Noise

In order to estimate transmission capacity using WGN, an optical WGN signal is required. Here, we use a combination of two erbium-doped fiber amplifiers (EDFAs) and an optical bandpass filter to generate a 150 GHz wide Gaussian noise signal, as shown in Fig. 1. The WGN signal is split into a reference path (**x**) and a test path (**y**). A second 350 GHz wide noise band (**z**) is generated and added to the test path to vary the optical signal-to-noise ratio (OSNR). The reference and test path are delayed with respect to each other such that they can be received using the same receiver[6], see Fig. 2, and a variable optical attenuator (VOA) in the reference path allows to equalize the resulting electrical swing of the test and reference path. For capacity estimation of a transmission system, the system under test should be included in the test path and the noise loading stage might not be required. Here, we also delayed the coherent receiver local oscillator (LO) in order to reduce phase noise between the test and reference path, but this is not mandatory as shown in[4].

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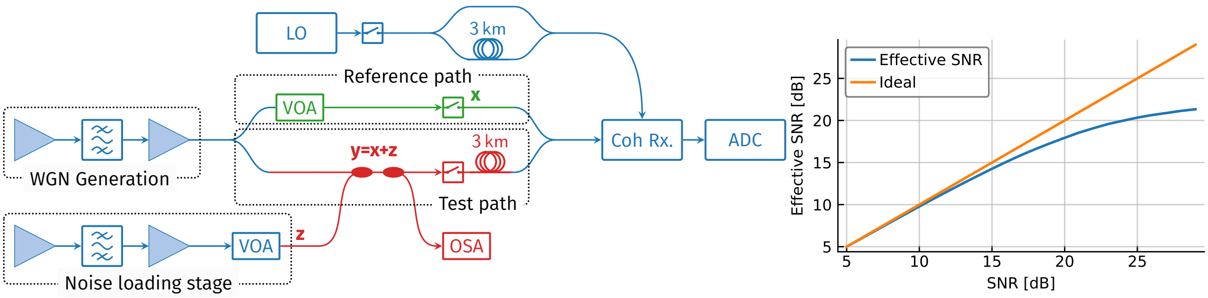

Fig. 1: (left) Experimental setup used for WGN based capacity estimation. A noise band is generated and split to create a reference path **x** and a test path **y**. Noise **z** is added to the test path such that **y** = **x**+**z**. The test and reference path are delayed with respect to each other such that they can be detected using a single receiver. **(right)** Plot of effective SNR based on received signals and SNR obtained from OSNR measurements.

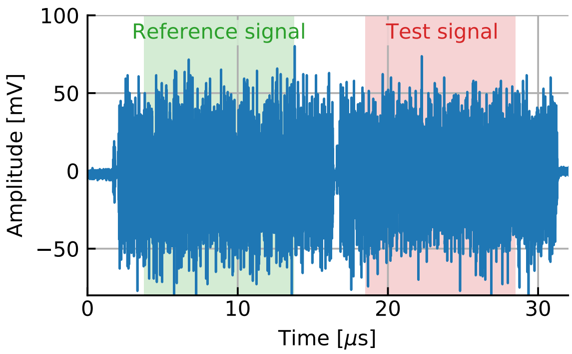

Fig. 2: Trace from the analog-to-digital converter (ADC) showing the time delayed reference and test signal for the in-phase component of the received X-polarization.

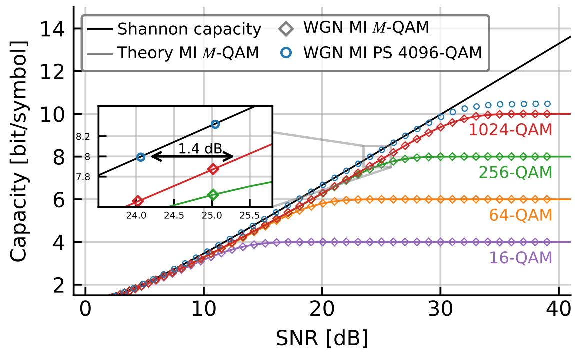

Fig. 3: Simulation results for AWGN channel showing capacity based on WGN estimation employing uniform QAM and PS 4096-QAM. A good fit between the theoretical *M*-QAM capacity and the WGN based capacity is observed. The PS 4096-QAM capacity is shown to follow the Shannon capacity.

After digitization by a 4 channel 80 GSa/s ADC with a bandwidth of 36 GHz, the following DSP steps are performed in order to estimate the transmission capacity:

1. The reference and test signal are extracted from the captured trace (see Fig. 2) and aligned in time using cross-correlation.
2. The reference signal is filtered to the desired test bandwidth, which we chose to be 20 GHz (resulting in an optical bandwidth of 40 GHz). Next, the reference signal is downsampled and treated as a 40 GBd signal, while the test signal is treated as the 2 times oversampled version of that signal.
3. Using a data-aided least-mean-squares equalizer, the test signal is equalized to the reference signal.
4. Noise realization **z** is estimated (**z'**) by subtracting the reference signal from the equalized test signal.
5. A signal (**x'**) is created containing symbols from the constellation that is used for WGN capacity estimation, as explained in the next section. **x'** is scaled such that it has the same power as the received reference signal.
6. Estimated noise realization **z'** is added to **x'**, such that **y'** = **x'**+**z'**
7. Capacity is estimated by calculating the mutual information (MI)[7] between **x'** and **y'**.

Designing a PS constellation based on WGN

The capacity obtained by the WGN based estimation method is dependent on the modulation format used to generate **x'**. This can be seen in the Fig. 3, where the WGN based capacity estimation method is simulated with different order square QAM formats. As can be seen, the plotted WGN MI and the theoretical *M*-QAM MI perfectly overlap, indicating the good performance of the WGN based estimation method. However, from the inset in Fig. 3 it is observed that there is a gap between the estimated capacity using *M*-QAM and the Shannon capacity, related to the shaping gain[1].

Since the optimum PS constellation is dependent on the SNR and has a Gaussian distribution for the AWGN channel, we use the reference signal to design a PS constellation for each captured trace used for WGN based capacity estimation.

The PS constellation is designed by first mapping the noise samples from the reference signal **x** to constellation points from a carefully

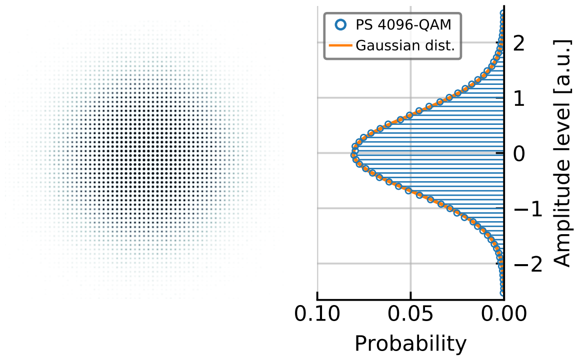

Fig. 4: (left) PS 4096-QAM constellation obtained from noise in reference path, for SNR = 40 dB. **(right)** Amplitude distribution of a single dimension.

scaled *M*-QAM constellation (here we used $M = 4096$) with the smallest Euclidean distance. This results in a probabilistically shaped signal and constellation, as the reference signal has a Gaussian distribution. Note it is paramount to match the power of the generated probabilistically shaped signal **x'** to the reference signal **x**. To this end, the power of **x'** is compared with **x** to adjust the scaling factor of the *M*-QAM constellation. This process is repeated to optimize the scaling factor, iteratively matching the power of **x'** to **x**. This results in a constellation as shown in Fig. 4, from where it can be seen that the constellation follows a Gaussian distribution. The obtained shaped signal is now used as **x'** for further processing.

The designed PS 4096-QAM constellations are used for WGN based capacity estimation, and simulation results are depicted in Fig. 3. It is observed that the estimation based on the PS 4096-QAM follows the Shannon capacity curve, indicating that the designed PS constellations are optimum.

Experimental validation

Using the method described above, we experimentally validated the WGN based capacity estimation method, of which the results are given in Fig. 5. Here, we estimate the SNR by converting the OSNR measured using an optical spectrum analyzer (OSA). It can be seen that for 16-QAM, the WGN capacity has a close fit with the theoretical capacity of 16-QAM, however for 64-QAM and 256-QAM the capacity is underestimated. This can be explained by the influence of the receiver SNR on the estimation of **x** and **y**. Fig. 1 (right) depicts the effective SNR based on the output of the equalizer and the SNR obtained from the OSNR. It is seen that starting from a SNR of 10 dB, the effective SNR starts

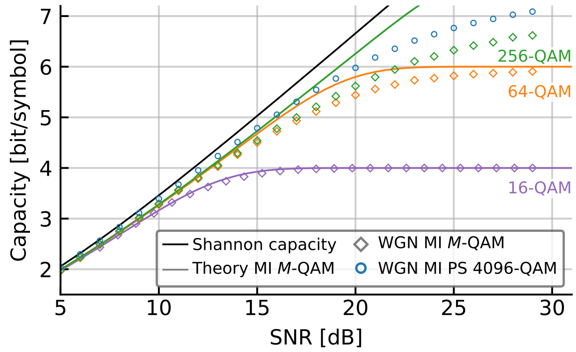

Fig. 5: Experimental results showing transmission capacity calculated based on WGN. It can be seen that the designed PS 4096-QAM constellation is closest to the Shannon capacity. A gap between the estimated capacity and the Shannon capacity is observed and can be explained by receiver noise limiting the effective SNR (see Fig. 1).

to drop with respect to the ideal SNR due to the impact of noise sources in the receiver. Hence, the WGN estimation method is estimating the capacity of the test path in combination with the receiver in that regime.

From Fig. 5, it can also be observed that the estimation using the PS 4096-QAM modulation format is the closest to the Shannon capacity and approaching the Shannon capacity for low SNR where the effective SNR equals the SNR. This indicates that the design of the PS 4096-QAM modulation using the experimentally obtained noise approaches an optimum constellation.

Conclusions

We demonstrate the use of white Gaussian noise (WGN) in combination with probabilistic constellation shaping in order to estimate transmission link capacity. We show it is paramount to carefully choose and optimize the constellation to be used with WGN based capacity estimation. Using the proposed method, the generated probabilistically shaped constellations are shown to approach Shannon capacity in simulation. Furthermore, it is shown experimentally that the probabilistically shaped constellation is closest to the Shannon capacity. However, as the WGN based capacity also includes receiver noise sources in the capacity estimation, a gap is observed between the estimated and Shannon capacity.

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