Photon-Efficient Communication Based on BPSK Modulation with Multistage Interferometric Receivers

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Abstract *Transmission of BPSK words optically decoded using a cascade of interferometric stages presents an alternative to current modulation standards for photon-starved communication, shifting the bulk of the system complexity to the receiver. Importantly, imperfect interference visibility does not undermine the attainability of high photon efficiency.*

Introduction

In specialized optical communication scenarios, such as downlink transmission of data collected by deep-space probes[1], the typical power spectral density (PSD) of the received optical signal remains well below the energy of a single photon at the carrier frequency per unit time-bandwidth area. Compared to conventional fiber optical communications[2], such photon-starved regime requires distinct modulation/demodulation strategies to maximize the information rate. Under severe power constraints, the relevant figure of merit to characterize the system performance is the photon information efficiency (PIE) specifying how much information can be carried by one received photon[3]. The current modulation standard for deep-space optical communication links is the pulse position modulation (PPM) format for which the PIE value of 13 bits per photon has been demonstrated[4] with photon counting direct detection. Scaling up the PIE requires increasing the modulation format order, which in the case of the PPM format translates into a higher peak-to-average power ratio for the emitted optical signal. The latter, however, is typically constrained by the physics of the transmitter laser subsystem. Other methods that avoid high peak-to-average power ratio, such as frequency shift keying (FSK), rely on a more intricate construction of the transmitter[5]. A recently proposed alternative[6] addressing this issue is to use binary phase shift keyed (BPSK) optical signal with suitable encoding. Properly chosen words composed from elementary BPSK symbols can be converted optically into the PPM format in the receiver using a cascade of interferometric stages[7] that concentrate the signal optical energy in the optical domain. In the ideal case the resulting PIE is equivalent to that of PPM, but attained with uniformly distributed optical energy in the transmitted signal. Moreover, the format order can be increased simply by generating longer BPSK words.

The purpose of this paper is to assess the impact of non-ideal interference visibility in the multistage receiver for the above scenario. It is found that the PIE scaling with the signal PSD remains the same as in the ideal case, while the actual PIE value is reduced by a multiplicative factor that can be estimated solely using the interference visibility. This is rather propitious given that the number of interferometric stages increases with the format order, hence potentially compounding errors induced by non-ideal interference.

Modulation and optical decoding

The method for photon-efficient communication considered here uses sequences of $M = 2^m$ BPSK pulses occupying individual temporal slots. Each sequence is modulated as one of M words defined by rows of an M-dimensional orthogonal Hadamard matrix[6],[8]. The words can be conveniently labeled with m-bit strings $\mathbf{b} = b_{m-1}\ldots b_1 b_0$. The ith bit, $i = m-1,\ldots,1,0$, contributes a phase factor alternating between 1 and $(-1)^{b_i}$ every 2^i slots. The overall phase of an individual pulse in the sequence is given by a product of the phase factors contributed by all the bits in the string $\mathbf{b}$. For example, Hadamard words for $M = 8$ are explicitly given by $\big(1, (-1)^{b_0}, (-1)^{b_1}, (-1)^{b_1+b_0}, (-1)^{b_2}, (-1)^{b_2+b_0}, (-1)^{b_2+b_1}, (-1)^{b_2+b_1+b_0}\big)$.

Optical decoding of the Hadamard words is implemented using a cascade of m interferometric stages. Each stage acts as a 50:50 beam splitter on the optical field in pairs of adjacent time intervals, as shown in Fig. 1(a). Such a transformation can be realized e.g. using fast polarization switching and polarization-dependent delay

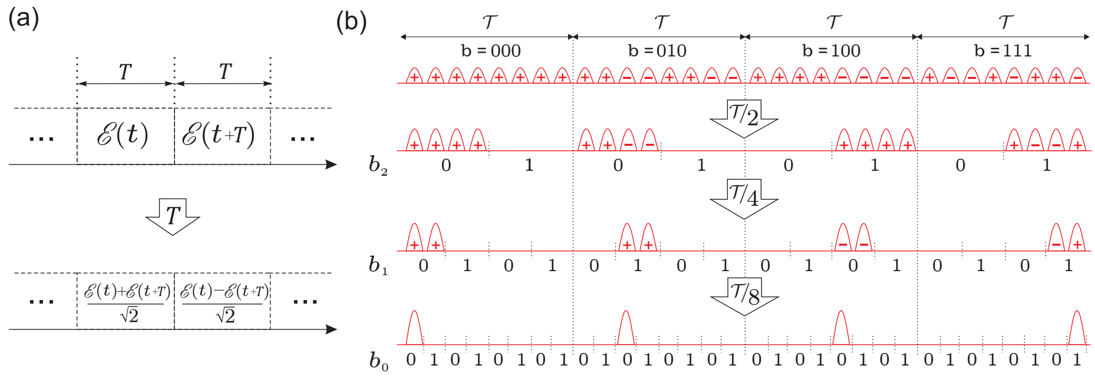

Fig. 1: (a) One stage of the interferometric receiver, represented as a wide vertical arrow, implements a 50:50 beam splitter transformation between pairs of adjacent time intervals. The label T denotes the interval duration. (b) A cascade of $m = 3$ interferometric stages processing BPSK Hadamard words of length $M = 2^m = 8$. Consecutive stages act on time intervals $\mathcal{T}/2$, $\mathcal{T}/4$, $\mathcal{T}/8$ that are power of two fractions of the sequence duration $\mathcal{T}$. Each stage concentrates the optical energy in one of the two input intervals that is determined by the value of the respective bit in a string $\mathbf{b} = b_2b_1b_0$ labeling the Hadamard word.

lines[7]. The length of the time interval for the jth stage, $j = 1, 2, \ldots, m$, is set to $\mathcal{T}/2^j$, where $\mathcal{T}$ is the duration of the entire sequence. Because of the mathematical construction of Hadamard matrices, each stage will concentrate the entire optical energy in one of the two paired input intervals. That way the jth stage optically decodes the value of the bit b_{m-j} as illustrated with Fig. 1(b) for sequences of length $M = 8$ processed by $m = 3$ stages. Eventually, the cascade concentrates the entire optical energy of the sequence in a single temporal slot related one-to-one with the input Hadamard word. The position of that slot is determined by direct detection.

When the interference visibility V of individual receiver stages is less than one, $V < 1$, only a fraction $\frac{1}{2}(1+V)$ of the optical energy is concentrated in the correct time interval, while the remaining part $\frac{1}{2}(1-V)$ is left in the adjacent interval. As a result, at the output of the cascade the optical energy is distributed unevenly between multiple slots as shown in Fig. 2. In such a general case the attainable information rate is given by the Shannon mutual information between the input Hadamard words $\mathbf{b}$ and photon counting events detected at the cascade output.

Because of the photon counting detection technique, it is useful to start by analyzing the transformation of the optical signal in the multistage receiver for a well defined total photon number contained in the pulse sequence. When the input sequence carries exactly one photon in total, the factors $\frac{1}{2}(1 \pm V)$ specify the probabilities that the photon is routed either into the correct or incorrect time interval. Formally, incorrect routing corresponds to flipping the value of the bit supposed to be decoded at a given stage. Hence the operation of a non-ideal multistage receiver for a single input photon can be described as a composition of $m = \log_2 M$ binary symmetric channels, each one with the same error probability $\frac{1}{2}(1-V)$. Consequently, the Shannon mutual information for one photon occupying the sequence reads $\left[1 - \mathrm{H}\left(\frac{1}{2}(1-V)\right)\right] \log_2 M$, where $\mathrm{H}(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary entropy function. When two or more photons are present in the input sequence, non-ideal interference may route them into different slots at the output of the cascade, producing photocounts in multiple slots within the sequence duration. Under the simple decoding assumption, such occurrences are treated as erasures and only events when all the photons are detected in the same slot are retained. Furthermore, it will be assumed that routing of multiple photons through the cascade is statistically independent. This model is motivated by a scenario where photons arrive in different spatial modes e.g. due to atmospheric turbulence[9],[10].

Optimization

The attainable PIE is a function of the PSD of the received optical signal, which can be characterized by the mean photon number per slot n_s. For a given n_s the PIE needs to be optimized over the format order M. In the case of ideal interference, $V = 1$, and Poissonian photon number statistics, the maximum attainable PIE, expressed in bits per photon, is well approximated by the expression[8]

$$\Pi(n_s) = \left(W\left(\frac{2e}{n_s}\right) - 2 + \left[W\left(\frac{2e}{n_s}\right)\right]^{-1} \right) \log_2 e \tag{1}$$

where $W(\cdot)$ is the Lambert function and e is Euler's number. This expression is depicted in Fig. 3 along with results of numerical optimization over

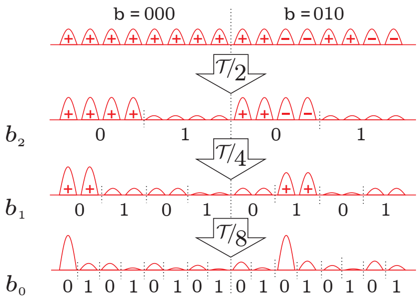

Fig. 2: When the interference visibility V of an individual receiver stage is less than one, a fraction $\frac{1}{2}(1+V)$ of the input optical energy is transferred to the correct time interval, while the remaning part $\frac{1}{2}(1-V)$ is left in the other interval that corresponds to the flipped value of the respective bit.

M restricted to integer powers of 2.

In a realistic scenario with non-unit interference visibility V the PIE has been calculated numerically using formulas given in Appendix and optimized over M taken as an integer power of 2. It has been assumed that the detector used at the cascade output does not have photon number resolving capability. The results are shown in Fig. 3. It is seen that the PIE increases unboundedly with diminishing n_s even when the interference visibility is substantially less than one.

An approximate analytical formula can be derived from a simplifying assumption that only cases when the received pulse sequence contains exactly one photon contribute to the mutual information. Such cases occur with the probability $p_1 = (Mn_s)\exp(-Mn_s)$. Following discussion presented in the preceding section, the effective PIE would be $p_1/(Mn_s)\left[1 - \mathsf{H}\left(\frac{1}{2}(1-V)\right)\right]\log_2 M$. Expanding p_1 up to the second order in Mn_s allows one to express the maximum as

$$\mathsf{PIE}^* \approx \left[1 - \mathsf{H}\left(\tfrac{1}{2}(1-V)\right)\right]\Pi(2n_s), \qquad (2)$$

where $\Pi(\cdot)$ has been defined in Eq. (1). As seen in Fig. 3, this formula slightly underestimates the value obtained from numerical optimization.

Conclusions

Photon-efficient communication can be realized with a BPSK signal using encoding based on Hadamard words that are optically decoded in a cascade of interferometric modules. The use of the BPSK signal may be beneficial in scenarios where the simplicity and adaptability of the transmitter module are of primary concern. An example of such a scenario is highly asymmetric communication with deep-space probes, where most of information transmission takes place space-to-ground. In contrast to the requirements for the transmitter module, the size, weight, and power

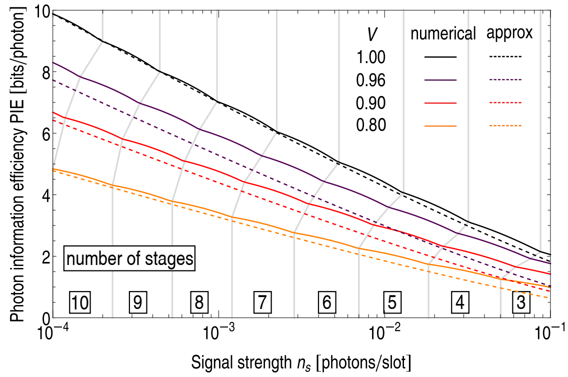

Fig. 3: Photon information efficiency PIE as a function of the signal strength quantified with the photon number per slot n_s for selected visibilities V of the interferometric stages. Results of numerical optimization over the format order M restricted to integer powers of 2 (solid lines) are compared with approximations (dashed lines) given by Eq. (1) for $V = 1.00$ and Eq. (2) otherwise. Light solid transverse lines separate regions of different optimal format order M^* labeled with boxes specifying the number of stages $m^* = \log_2 M^*$.

consumption characteristics of the receiver subsystem are not critical and it remains accessible on the ground for maintenance and upgrades.

Practical construction of a multistage interferometric receiver presents a number of technical challenges, including signal synchronization and loss suppression. While the signal loss can be taken into account by simple rescaling of the received signal PSD, results presented here provide a simple way to estimate the impact of imperfect interference visibility on the receiver performance. Importantly, effects of atmospheric turbulence can be mitigated using delay line interferometers accepting multiple spatial modes[9],[10].

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Appendix

It is convenient to label the output slots with binary strings $\mathbf{c} = c_{m-1}\ldots c_1 c_0$ and define the Hamming distance $d(\mathbf{b},\mathbf{c}) = \sum_{i=0}^{m-1} b_i \oplus c_i$. The conditional probability of an exclusive photocount event in a slot $\mathbf{c}$ given the input word $\mathbf{b}$ reads

$$p_{\mathbf{c}|\mathbf{b}} = \sum_{k=0}^{\infty} p_k \left(\frac{1+V}{2}\right)^{k[m-d(\mathbf{b},\mathbf{c})]} \left(\frac{1-V}{2}\right)^{kd(\mathbf{b},\mathbf{c})} \qquad (3)$$

where k is the photon number in the sequence taken to follow the Poisson distribution $p_k = \exp(-Mn_s)(Mn_s)^k/k!$. The set of conditional probabilities $p_{\mathbf{c}|\mathbf{b}}$ has been used to evaluate numerically the mutual information for uniformly distributed input words, $p_{\mathbf{b}} = 1/M$. The sum over k in Eq. (3) has been truncated at 10 photons.

References

[1] H. Hemmati, A. Biswas, and I. B. Djordjevic, "Deep-space optical communications: Future perspectives and applications", *Proc. IEEE*, vol. 99, no. 11, pp. 2020–2039, 2011. DOI: 10.1109/JPROC.2011.2160609.

[2] R.-J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, "Capacity limits of optical fiber networks", *J, Lightwave Technol.*, vol. 28, no. 4, pp. 662–701, 2010. DOI: 10.1109/jlt.2009.2039464.

[3] H. Hemmati, Ed., *Deep-Space Optical Communications*. John Wiley & Sons, Inc., 2006.

[4] W. H. Farr, J. M. Choi, and B. Moision, "13 bits per incident photon optical communications demonstration", in *Free-Space Laser Communication and Atmospheric Propagation XXV*, H. Hemmati and D. M. Boroson, Eds., International Society for Optics and Photonics, vol. 8610, SPIE, 2013, pp. 30–38. DOI: 10.1117/12.2007000.

[5] S. J. Savage, B. S. Robinson, D. O. Caplan, J. J. Carney, D. M. Boroson, F. Hakimi, S. A. Hamilton, J. D. Moores, and M. A. Albota, "Scalable modulator for frequency shift keying in free space optical communications", *Opt. Express*, vol. 21, no. 3, pp. 3342–3353, 2013. DOI: 10.1364/OE.21.003342.

[6] S. Guha, "Structured optical receivers to attain superadditive capacity and the Holevo limit", *Phys. Rev. Lett.*, vol. 106, no. 24, p. 240 502, 2011. DOI: 10.1103/PhysRevLett.106.240502.

[7] K. Banaszek and M. Jachura, "Structured optical receivers for efficient deep-space communication", in *2017 IEEE International Conference on Space Optical Systems and Applications (ICSOS)*, Nov. 2017, pp. 34–37. DOI: 10.1109/ICSOS.2017.8357208.

[8] K. Banaszek, L. Kunz, M. Jachura, and M. Jarzyna, "Quantum limits in optical communications", *J. Lightwave Technol.*, vol. 38, no. 10, pp. 2741–2754, 2020. DOI: 10.1109/JLT.2020.2973890.

[9] Z. Sodnik and M. Sans, "Extending EDRS to laser communication from space to ground", in *International Conference on Space Optical Systems and Applications (ICSOS) 2012*, Oct. 2012, pp. 13–2.

[10] J. Jin, J.-P. Bourgoin, R. Tannous, S. Agne, C. J. Pugh, K. B. Kuntz, B. L. Higgins, and T. Jennewein, "Genuine time-bin-encoded quantum key distribution over a turbulent depolarizing free-space channel", *Opt. Express*, vol. 27, no. 26, pp. 37 214–37 223, 2019. DOI: 10.1364/OE.27.037214.