Impact of Sparse Gain Equalization in the Presence of Stimulated Raman Scattering

Chiara Lasagni, Paolo Serena, Alberto Bononi

University of Parma, Department of Engineering and Architecture, Parma, Italy. chiara.lasagni@unipr.it

Abstract *We investigate the impact of accumulated stimulated Raman scattering (SRS) on the nonlinear interference (NLI) variance due to sparse gain equalization along the link. We propose simple modifications to analytical models from the literature for reliable NLI estimation in the presence of accumulated SRS.*

Introduction

Inter-channel stimulated Raman scattering (SRS) is a wideband nonlinear process for which higher frequencies are depleted while amplifying lower frequencies, yielding a tilted signal power profile[1]. Several efforts have been made in the literature to model the impact of SRS on the received signal. An extension of the Gaussian noise (GN) model[2] to include inter-channel SRS in the estimation of the nonlinear interference (NLI) variance was proposed in integral form[3]–[5], and in closed-form[6],[7] for the self-phase modulation (SPM) and the cross-phase modulation (XPM) contributions.

The modulation format has been later included in the integral form in[8],[9] through a modified enhanced GN (EGN) model[10], or nonlinear interference noise (NLIN) model[11], and in closed-form formulas by a correction term to XPM only[12]. Unfortunately, these models assume that the SRS gain tilt is perfectly equalized after each span through a dynamic gain equalizer (DGE) or gain-flattening filter, while real systems normally include DGEs only every few spans[13].

In this work, we show that such a sparse positioning of DGE has serious implications on the models' accuracy. We show a simple yet reliable approximation to overcome such difficulties in both the EGN model and the closed-form expressions of[7].

Inclusion of accumulated SRS

An ideal DGE equalizes the SRS gain tilt by perfectly flattening the power spectrum. Fig. 1 sketches an example where a DGE is placed every $N_s = 3$ spans for a total of $N_D = 2$ DGEs. The bottom part of the figure shows an example of the evolution of the power spectral density (PSD) in the first link section to highlight the effects of SRS accumulation through spans.

Over the C+L band, the PSD tilt is usually modeled through the triangular approximation of the Raman gain[14]. For a link having an ideal DGE and amplifier after each span, the gain/loss at frequency f is:

$$\rho(z, f) = \Upsilon(z) e^{-PC_r L_{\mathrm{eff},z} f} e^{-\alpha z} \tag{1}$$

where z is the local coordinate within a span, α the fiber attenuation coefficient, $L_{\mathrm{eff},z} = \frac{1-e^{-\alpha z}}{\alpha}$ is the effective length up to z, P the total wavelength division multiplexing (WDM) power, and C_r is the SRS coefficient of the triangular approximation. We defined the factor Υ as

$$\Upsilon(z) \triangleq \frac{P}{\int G_{\mathrm{TX}}(\nu) e^{-PC_r L_{\mathrm{eff},z}\nu} \mathrm{d}\nu} \tag{2}$$

where $G_{\mathrm{TX}}(\nu)$ is the transmitted PSD at frequency ν. The term Υ plays the role of a normalization factor[14] ensuring that the total power is lost only through the fiber attenuation.

This SRS-modified power profile can be included in the span kernel to weight the four-wave mixing (FWM) efficiency:

$$\eta_1(f, f_1, f_2) = \int_0^L \rho(z, f + f_1 + f_2) e^{j\Delta\beta z} \mathrm{d}z \tag{3}$$

where L is the span length, and $\Delta\beta = 4\pi^2 f_1 f_2[\beta_2 + \pi(f_1 + f_2 + 2f)\beta_3]$ is the phase-matching coefficient[2]. Closing the integral in z is of paramount importance for an efficient implementation of any GN-based model, as well as searching for closed-form formulas, since the oscillating behavior of the integrand calls for many

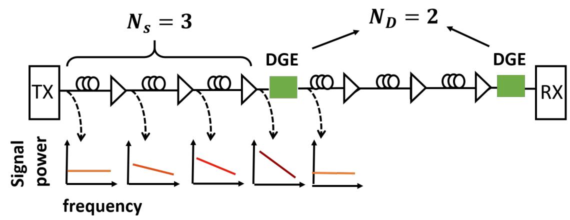

Fig. 1: Sketch of a 6 span link with lumped amplification and gain-tilt equalization by $N_D = 2$ DGEs with $N_s = 3$ spans between them.

integration points along z. A reliable approximation to close the integral has been shown in[7], based on the observation that within a span the SRS is small such that the integrand can be expanded in a first-order Taylor series with respect to $PC_r L_{\text{eff},z}$.

However, when the DGE is not placed after each span, the term $L_{\text{eff},z}$ in the kth span between two neighboring DGEs must be substituted to account for the accumulated SRS as

$$L_{\text{eff},z} \rightarrow L_{\text{eff},z} + (k-1)L_{\text{eff},L} \tag{4}$$

where $k \in (1, \ldots, N_s)$. Unfortunately, such generalized $L_{\text{eff},z}$ may not be small anymore, invalidating the first-order Taylor series approximation. As a direct consequence, the z-integral of the kernel cannot be closed and requires to be evaluated through numerical integration. Hence, we refer to this model as the *integral model*.

Simplified model and closed-form

The SRS-induced tilt gets big because of the accumulated $L_{\text{eff},L}$ between DGEs, plus a disturbing L_{eff}, z term that does not allow to close the integral in z. Our key idea is to simplify the model by removing the dependence on z in the normalization factor only, by forcing in it $L_{\text{eff},z} \approx 0.5 \cdot L_{\text{eff},L}$ obtaining $\Upsilon_k(z) \approx \Upsilon_k$.

Thanks to this approximation in the normalization factor, all the span-dependent terms can now be factored out of the z-integral allowing to express the link kernel with the product of an inter-span dependent term and an intra-span term:

$$\eta(f, f_1, f_2) \approx \chi(f, f_1, f_2)\eta_1(f, f_1, f_2). \tag{5}$$

The inter-span term χ, which reduces to a phased-array term in the absence of accumulated SRS, takes the following novel expression when accumulated SRS is considered:

$$\chi(f, f_1, f_2) \triangleq \sum_{d=1}^{N_D} e^{j\Delta\beta(d-1)LN_s} \sum_{k=1}^{N_s} e^{j\Delta\beta(k-1)L} \times e^{-PC_r(k-1)L_{\text{eff},L}(f+f_1+f_2)}\Upsilon_k. \tag{6}$$

The intra-span term η_1 in Eq. (5) can now be closed with the same first-order Taylor approximation of[7], while the summation over k underpinning the inter-span term cannot. However, since N_s is usually small, its numerical computation takes a few seconds. We call the model resulting from this approximation the *simplified model*, which differs from the model in the absence of accumulated SRS[7] only in the term χ. Such a modified kernel can be used in the EGN without further modifications in the algorithm[8]. It can also be used to modify simple scaling expressions with the number of spans usually adopted in the literature[2]. In particular, focusing on the GN-term, after N spans the SPM and XPM variance for the ith channel can be expressed as

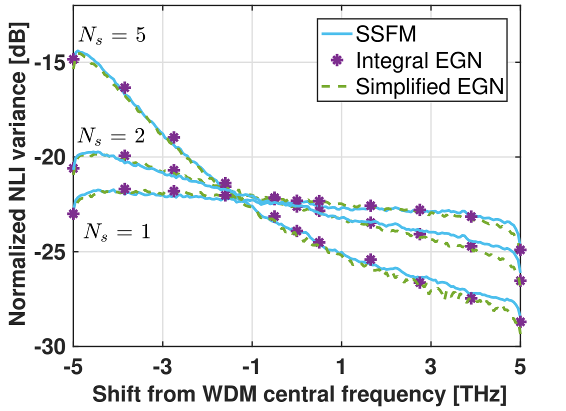

Fig. 2: Normalized NLI variance (1/mW^2, expressed in dB) vs the frequency shift from the WDM central frequency. PDM-64QAM, 201 channels, 10 $\times$ 100 km with DGE period $N_s = 1$, 2 or 5 spans. Integral EGN: EGN model with numerical integration along the distance. Simplified EGN: EGN model with the proposed modification in Eq. (6).

$$\sigma^2_{\text{SPM},N}(i) = \mathcal{N}_i^{1+\varepsilon}\, \sigma^2_{\text{SPM},1}(i) \tag{7}$$

$$\sigma^2_{\text{XPM},N}(i) = \sum_{\ell \neq i} \mathcal{N}_\ell\, \sigma^2_{\text{XPM},1}(i, \ell) \tag{8}$$

where $\sigma^2_{\text{SPM},1}(i)$ and $\sigma^2_{\text{XPM},1}(i, \ell)$ are the single-span variances due to SPM and the ℓth interfering channel XPM, respectively. Closed-form expressions for the single-span variances can be found in[7]. The coherency factor ε is defined as in[2] and accounts for the coherent accumulation of NLI along the link. The scaling factor is defined as $\mathcal{N}_\ell = |\chi|^2_{\text{inc}} \approx |\chi(0, 0, f_\ell)|^2_{\text{inc}}$ where f_ℓ is the central frequency of channel ℓ, and the subscript *inc* indicates the incoherent contribution[2]. In the absence of accumulated SRS[2],[7] it is $\mathcal{N}_\ell = N$, while in our extended model with DGE every N_s spans it writes as

$$\mathcal{N}_\ell = N_D \sum_{k=1}^{N_s} e^{-2PL_{\text{eff},L}C_r f_\ell (k-1)} \Upsilon_k^2. \tag{9}$$

Hence, the closed formulas in[7] can be used as well by substituting the new expression of $\mathcal{N}_\ell$.

Numerical results

We validated the proposed extended EGN models against split-step Fourier method (SSFM) simulations. Each Kerr effect (SPM, XPM, and FWM-based terms[8]) has been evaluated with the expressions discussed in[10] through Monte Carlo estimations[8],[11] with the new modified

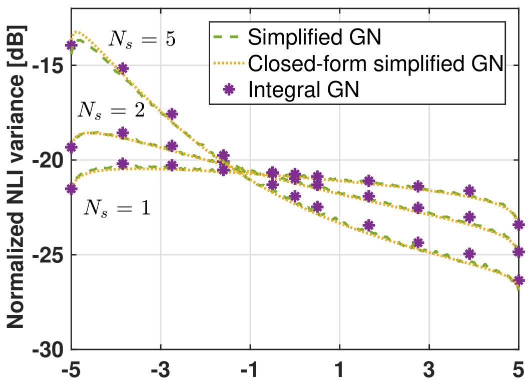

Fig. 3: Normalized NLI variance vs frequency shift. Same data as Fig. 2, but Gaussian distributed symbols. Closed-form refers to the formula[7] modified with Eq. (9).

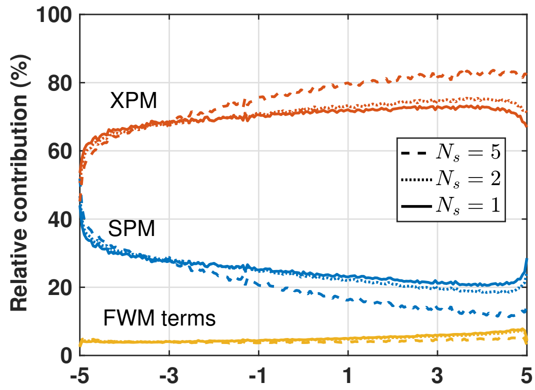

Fig. 4: Relative contribution of SPM, XPM, and FWM to the overall NLI variance in Fig. 2 vs frequency shift for different DGE periods.

kernels. The links under test are dispersion-uncompensated links, composed of single-mode fibers (SMFs) and based on lumped amplification. Since modeling the NLI is our target, we considered noiseless amplifiers whose gain perfectly recovered the loss due to fiber attenuation. The SRS gain on the signal power was periodically compensated by ideal DGEs, whose number within the link was varied. The SMFs had length $L = 100$ km, attenuation $\alpha = 0.2$ dB/km, dispersion $D = 17$ ps/nm/km, dispersion slope $S = 0.057$ ps/nm^2/km, and nonlinear coefficient $\gamma = 1.26$ 1/W/km. The slope of the linear approximation of the Raman gain of the fiber was $C_r = 0.028$ 1/THz/km/W. We transmitted polarization division multiplexing (PDM) signals with modulation format 64 quadrature amplitude modulation (QAM).

We considered a WDM bandwidth of 10 THz, filled with 201 channels spaced 50 GHz, having symbol rate 49 Gbaud, root-raised cosine supporting pulses with roll-off 0.01, and power 0 dBm. Fig. 2 shows the NLI variance, normalized to the cube of the channel power, as a function of the frequency shift from the WDM central frequency, for a 10 span link with DGEs placed each 1, 2 or 5 spans. We used solid lines for the SSFM results, markers for the integral EGN model, and dashed lines for the simplified model.

Due to the distributed interaction between SRS and Kerr effects, the NLI variances in Fig. 2 exhibit tilted profiles. Most important, the tilt is emphasized by the accumulation of SRS between DGEs, yielding curves with $N_s > 1$ far apart from the benchmark N_s=1 case usually analyzed in the literature. It can be seen that both the integral and the simplified EGN model proposed in this work correctly estimate the impact of the accumulated SRS on the NLI, with an average error across the WDM bandwidth less than 0.25 dB compared to SSFM results.

The simplified EGN model, besides accurate, is also very quick to compute, requiring less than 1 minute to evaluate the NLI variance per channel, while the integral EGN required $\approx$ 50 min.

We also checked the closed-form expressions[7] with the novel $\mathcal{N}$ in Eq. (9). Since these closed-forms are valid only for the GN-part of the NLI variance, we substituted the QAM with Gaussian distributed complex symbols. All the other transmission parameters are those of Fig. 2. In Fig. 3 it can be seen that the average gap across the WDM bandwidth between the closed-form expression and its Monte Carlo counterpart is $\approx$ 0.1 dB. It should be noted that both the integral and the simplified model account for all the Kerr effects, while the closed-forms only include SPM and XPM.

Finally, we investigated the impact of the number of DGEs on each nonlinear effect. Fig. 4 shows such contributions expressed in percentage to the overall NLI, obtained with the simplified model. It is worth noting that a reduced number of DGEs increases the importance of XPM at the expense of SPM in most of the WDM bandwidth, thus reducing the effectiveness of fiber nonlinearity equalization algorithms.

Conclusions

We showed that the number of DGEs along the link for the compensation of the SRS tilt has important implications on the received NLI variance. We proposed simple modifications to the EGN model and closed-form expressions available in the literature to preserve their accuracy.

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