

# Rate Adaptation by Low-Cost Complementary Mapper Shaping for Short-Reach IM-DD Systems

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**Abstract** We propose a low-cost probabilistic shaping method called the complementary mapper (CM) shaping, and experimentally demonstrate a wide range of rate adaptation using the CM shaping in an intensity modulation and direct detection (IM-DD) system with 80-Gbaud pulse-amplitude modulation (PAM).

## Introduction

Two unique characteristics should be considered to implement probabilistic shaping (PS) in short-reach intensity modulation (IM) and direct detection (DD) systems, which are distinguished from long-haul coherent systems. First, in the absence of optical amplifiers, IM-DD systems are subject to the peak power constraint of the electrical and optical components inside the transceiver rather than the average power constraint thereof. In such systems, reduction in average signal power by PS does not directly translate into enhanced system performance<sup>[1]</sup>. However, if digital signal processing (DSP) in the transmitter converts modulation symbols of *any* distribution into Gaussian-like signals, it has been revealed<sup>[2]</sup> recently that the peak power constraint becomes much less relevant, and the rate adaptability and shaping gain of PS can partially or even entirely be exploited (e.g., see Ref.<sup>[3]</sup>). The assumption on the use of transmitter DSP would be justified in many PS systems since they require an application specific integrated circuit (ASIC) in which various DSP can be incorporated to maximize the information rate (IR). Second, the implementation aspects of the PS, such as chip area, power consumption, and production cost constitute much more stringent requirements in short-reach systems than in long-haul systems. In particular, it is greatly beneficial to achieve a large cost saving in exchange for reduction in shaping gain of a few tenths of a decibel, provided that rate adaptability over a wide range of channel conditions is still ensured.

In this paper, we apply a PS method called the *complementary mapper (CM)* shaping to short-reach IM-DD systems, which is an extension of the cut-and-paste (CAP) shaping originally proposed in our earlier work<sup>[4]</sup>. The CM shaping is particularly suitable for cost-sensitive IM-DD systems owing to its low complexity via adding simple bit-to-amplitude mappers to a conventional system. We demonstrate through simulations and experiments that a widely rate-adaptable IM-DD system can be economically implemented by using the CM shaping.

## CM Shaping

We construct an  $M$ -PAM symbol by multiplying a sign and a positive amplitude, where the sign is uniformly distributed over  $\{\pm 1\}$  and the amplitude is drawn from alphabet  $\mathcal{A} = \{1, 3, \dots, M-1\}$  with unequal probabilities. To produce a non-uniform amplitude distribution, CM shaping encodes a length- $K$  bit sequence  $\mathbf{b} \in \{0,1\}^K$  into two length- $N$  amplitude sequences  $\mathbf{a} \in \mathcal{A}^N$  using two complementary bit-to-amplitude mappers  $\mathcal{M}_1$  and  $\mathcal{M}_2$ , and picks the one with a lower (or equal) energy, as shown in Fig. 1. Table 1(a) shows an example of the mappers  $\mathcal{M}_1$  and  $\mathcal{M}_2$  for  $M = 8$ ,  $K = 6$ ,  $N = 3$  where all possible input sequences  $\mathbf{b}$  are arranged in a lexicographical order, and the mappers  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are constructed such that the output amplitude sequences  $\mathcal{M}_1(\mathbf{b})$  and  $\mathcal{M}_2(\mathbf{b})$  have energies in ascending and descending orders, respectively. Namely, for any input sequences  $\mathbf{b}_i > \mathbf{b}_j$ ,  $\mathcal{M}_1$  fulfills  $\|\mathcal{M}_1(\mathbf{b}_i)\|^2 \geq \|\mathcal{M}_1(\mathbf{b}_j)\|^2$  and  $\mathcal{M}_2$  fulfills  $\|\mathcal{M}_2(\mathbf{b}_i)\|^2 \leq \|\mathcal{M}_2(\mathbf{b}_j)\|^2$ , where  $\|\cdot\|$  denotes the  $L_2$ -norm. Subsequently, the total energy is compared between  $\mathcal{M}_1(\mathbf{b})$  and  $\mathcal{M}_2(\mathbf{b})$ , and then the one with a smaller (or equal) energy is selected as the output of the CM shaping encoder. In the running example of Tab. 1(a), the low-energy amplitude

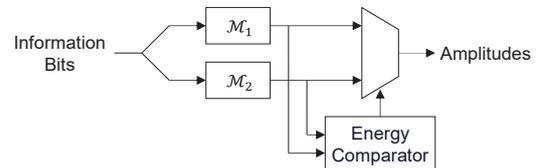


Fig. 1: CM shaping encoder

Tab. 1: Bit-to-symbol mapping for PAM-8

(a)  $K = 6, N = 3$

(b)  $K = 5, N = 3$

Input Data $\mathbf{b}$		Output Amplitudes		Input Data $\mathbf{b}$		Output Amplitudes	
Decimal	Binary	$\mathcal{M}_1(\mathbf{b})$	$\mathcal{M}_2(\mathbf{b})$	Decimal	Binary	$\mathcal{M}_1(\mathbf{b})$	$\mathcal{M}_2(\mathbf{b})$
0	000000	111	777	0	00000	111	553
1	000001	113	775	1	00001	113	535
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
62	111110	775	113	30	11110	535	113
63	111111	777	111	31	11111	553	111

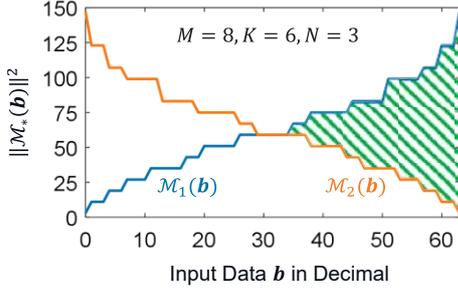


Fig. 2: Energy of amplitude sequences  $\mathcal{M}_1(\mathbf{b})$  and  $\mathcal{M}_2(\mathbf{b})$ .

sequence is simply  $\mathcal{M}_1(\mathbf{b})$  if  $\mathbf{b} \leq 31$  and  $\mathcal{M}_2(\mathbf{b})$  otherwise (cf. the block at the bottom of Fig. 1). Figure 2 shows the energies of all possible length- $N$  blocks of  $\mathcal{M}_1(\mathbf{b})$  and  $\mathcal{M}_2(\mathbf{b})$  for the running example. The area under the blue line in Fig. 2 is the energy consumed by a conventional uniform encoder that has only one mapper  $\mathcal{M}_1$  to encode all possible inputs  $\mathbf{b}$ , and the green shaded area shows the amount of the reduced energy by introducing another mapper  $\mathcal{M}_2$ . This amounts to an energy saving of 2.3 dB when averaged over all input data. In the running example, CM shaping produces amplitudes 1, 3, 5, 7 with probability of 0.39, 0.31, 0.23, 0.07, respectively, thereby realizing a Gaussian-like distribution when multiplied by equally probable signs in  $\{\pm 1\}$ .

The procedure illustrated above is the same as the CAP shaping<sup>[4]</sup>. While it produces a Gaussian-like distribution, its flexibility in rate adaptation is limited. To enhance its flexibility for IM-DD applications, we add two new features for the CM shaping as proposed in this work. First, after the bit-to-symbol mapping shown in Tab. 1, we divide each of the two successive streams from the two complementary mappers into blocks of  $D$  amplitudes (with  $D > 1$ ), and compare the total energy of the length- $D$  blocks rather than the length- $N$  blocks as above. This requires one *indicator* bit per each length- $D$  amplitude block to tell the receiver which of  $\mathcal{M}_1^{-1}$  or  $\mathcal{M}_2^{-1}$  should be used for demapping. The indicator bit is transmitted as a sign of one amplitude in each block and results in an overhead (OH) of 1 bit in each block. As  $D$  decreases (i.e., as the shaping OH ratio  $1/D$  increases), the average symbol energy decreases. In addition, we flexibly reduce  $K$  for a fixed  $N$ . For example, compared to Tab. 1(a), by reducing the input length  $K$  from 6 to 5 for the same output length  $N = 3$  as in Tab. 1(b), the IR can be reduced by a factor of  $5/6$  and more energy saving can be achieved. The mappers  $\mathcal{M}_1$  and  $\mathcal{M}_2$  should be modified correspondingly to fulfill the reverse-ordered energy relation.

The IR of the CM shaping in the probabilistic amplitude shaping (PAS) architecture<sup>[5]</sup> can be calculated as

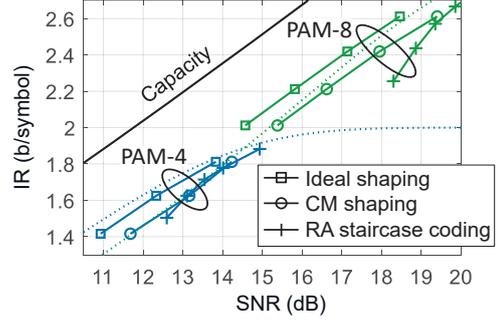


Fig. 3: IRs achieved by various rate adaptation schemes.

Tab. 2: CM shaping parameters used in the paper

$M$	$K$	$N$	$D$	IR
4	5	8	12	1.42
	7		8	1.63
	8		16	1.81
8	5	4	20	2.01
	6		10	2.21
	7		7	2.42
	8		5	2.61

$$IR = \underbrace{\frac{K}{N}}_{(a)} + 1 - \underbrace{m(1 - R_c)}_{(b)} - 1/D \quad (1)$$

in bit/symbol per dimension, where  $R_c$  is the rate of the underlying binary FEC code and  $m = \log_2 M$ . Here, the terms (a) and (b) quantify the amount of information carried by amplitude and sign, respectively. In term (b),  $m(1 - R_c)$  and  $1/D$  quantify the FEC OH and shaping OH, respectively, in each sign bit. As shown in Eq. (1), the CM shaping achieves high flexibility in rate adaptation by adjusting  $K$ ,  $N$ , and  $D$ , with low complexity by just adding an additional mapper and an energy comparator as in Fig. 1.

Figure 3 shows the IRs that can be practically achieved by CM shaping (circles) on PAM-4 (blue) and PAM-8 (green) templates using a 6.7%-OH hard-decision (HD) staircase code<sup>[6]</sup> in the additive white Gaussian noise (AWGN) channel. Here, the CM shaping produces the IRs from 1.42 to 2.61 b/symbol in increments of  $\sim 0.2$  b/symbol, using the parameters shown in Tab. 2. As a benchmark, also shown in Fig. 3 are the IRs achieved by ideal shaping using the 6.7%-OH staircase code (squares), uniform PAM with rate-adaptable (RA) staircase codes<sup>[7]</sup> (pluses), the capacity of the AWGN channel with real-valued input (black solid line), and the theoretically maximum achievable information rates (AIRs) of uniform PAM with bit-interleaved coded modulation (BICM) and HD-FEC (dashed lines). It can be seen from Fig. 3 that CM shaping (circles) achieves evenly-spaced IRs with a consistent gap to the capacity, whereas uniform PAM with RA staircase code (pluses) achieves non-uniform rate adaptability at inconsistent gaps to the capacity. Compared to ideal shaping, the CM shaping induces a performance penalty of

0.4 to 1 dB in SNR. Note however that rate adaptation by CM shaping is vastly simpler than by ideal shaping or RA-FEC.

### Experimental Setup

We evaluate CM shaping using an experimental IM-DD system shown in Fig. 4. PAM signals are generated at 80 GBaud using an 8-bit digital-to-analog converter (DAC) having 40-GHz analog bandwidth at 120-GSa/s. The electrical signals are boosted by a 55-GHz radio frequency (RF) amplifier to drive an O-band external modulated laser (EML). The EML, built by Source Photonics, consists of a distributed-feedback (DFB) laser and an electro-absorption modulator (EAM) with a 3-dB bandwidth of 32 GHz. Since the influence of chromatic dispersion is small in short-reach O-band systems, we evaluate the CM shaping in a back-to-back (B2B) configuration, with a variable optical attenuator (VOA) ahead of the receiver to emulate various link losses. The light is detected by a 70-GHz PIN photodiode (PD), amplified by a 60-GHz RF amplifier, and digitized by a 63-GHz real-time oscilloscope (RTO) at 160-GSa/s.

All the DSP is performed offline. At the transmitter, the 80-GBaud signals are band-limited by a root-raised cosine (RRC) filter with a roll-off factor of 0.05. As the cutoff frequency of RRC-pulse-shaped signals ( $40 \times 1.05 = 42$  GHz) significantly exceeds the 3-dB bandwidth of the EML (32 GHz), a strong frequency pre-emphasis filter is used to compensate for the power loss near the band edges. This transforms signals with various distributions into Gaussian-like signals, making the difference in peak-to-average power ratio (PAPR) less than 1 dB between the uniform and PS PAM signals, which could be greater than 6 dB otherwise. The fact that the PAPRs of uniform and PS PAM signals are rendered similar by sharp RRC filtering and strong frequency pre-emphasis implies that our IM-DD system is approximately subject to an average power constraint rather than a peak power constraint. At the receiver, signals are processed by matched filtering, timing recovery and 1024-tap least-mean-square (LMS) equalization with 2 samples per symbol.

### Experimental Results

Figure 5(a) shows the bit error rates (BERs) of CM-shaped PAM symbols (circles, solid lines), where five sets of the CM shaping parameters are chosen from Tab. 1. For a benchmark, also shown are the BERs of ideal shaping (squares, dotted lines) for the same IRs as the CM shaping, uniform PAM-4 (lower black solid line) and uniform PAM-8 (upper black solid line), and the BER threshold of  $4.8 \times 10^{-3}$  (red dashed line) required for error-free decoding with the 6.7%-

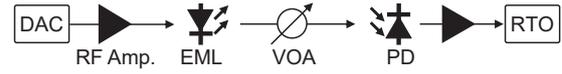


Fig. 4: Experiment setup.

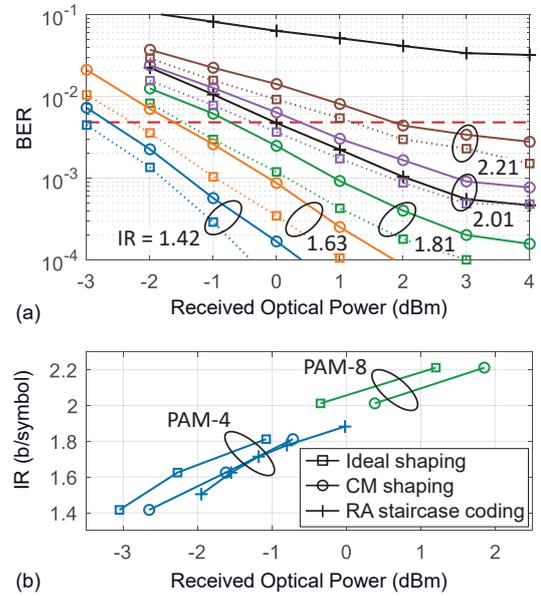


Fig. 5: (a) BER and (b) practically achievable IR of various rate adaptation schemes, as a function of the ROP.

OH HD staircase code<sup>[6]</sup>. From the BERs of Fig. 5(a), practically achievable IRs and their received optical power (ROP) to meet the BER threshold are estimated as in Fig. 5(b), for the CM (circles) and ideal (squares) shaping on PAM-4 (blue) and PAM-8 (green) templates. Fig. 5(b) also shows the IRs of uniform PAM-4 with RA staircase codes<sup>[7]</sup> (pluses). The IRs of uniform PAM-8 cannot be shown since no staircase code can succeed in error-free decoding due to the high BER floor of uniform PAM-8 after receiver DSP (cf. Fig. 5(a)). Clearly, Fig. 5 shows that CM shaping enables rate adaptation across a wide range of channel conditions in the experimental IM-DD system. Moreover, the CM shaping incurs sensitivity penalties of only 0.4 to 0.8 dB compared to ideal shaping and provides similar or better performance compared to RA staircase codes, despite the remarkably simpler description and implementation.

### Conclusions

We demonstrated rate adaptation by the CM shaping in an experimental IM-DD system. The low implementation complexity and good rate adaptability show that the CM shaping is well-suited for low-cost IM-DD systems.

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