Stochastic Anti-symmetric Schrodinger Equations for Non-Manakovian Propagation

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Abstract *Unpolarised and co-propagating ASE depolarizes a cw probe via the fibre's Kerr nonlinearity. We theoretically analyse these previous experimental observations of ultra-fast and PMD-dependent polarization state fluctuations. The probe's propagation does not obey the Manakov equation and is governed by an advanced Schrodinger formalism for addressing fibre PMD.*

Introduction

To deliver timely optimized system designs and cost-effective solutions, the undersea telecom industry strives for more accurate transmission modelling; now even scrutinizing weak acousto-optical channel interactions for next gen system designs[1]. Here, we theoretically analyse a recently observed transmission phenomenon that we refer as NL depolarization (NLDP) of light in fibre which can also slightly penalize a data signal. Un-polarised optical noise (ASE) rapidly changes the state of polarization (SOP) via the fibre's Kerr nonlinearity from a fully-polarized and co-propagating cw light by inducing anti-symmetric phase noise in both of its orthogonal polarization states.

This novel phenomenon has neither been predicted nor experimentally observed prior to its first reporting in 2019[2-4]. Meanwhile, the NLDP-induced SOP fluctuations have been experimentally characterized in the time[2,3] and frequency[4] domains, and in Stokes space[5]. While comparable small, it provides an excellent case study of effects that do not obey commonly-used transmission theory such as the Manakov equation (ME)[6]. The heuristically-determined ME has become an integral part of soliton theory and is often used in today's system modelling due to its relatively simple form and highly accurate results. Based on this success, attempts for deriving the ME from the more fundamental Coupled NL Schrodinger equations (CNLS), have been published[7]. We challenge them as they rely on an illusively compelling mathematical argument.

NLDP should not be confused with NL polarization rotation (NLPR)[8,9]. Essential for NLDP is a non-vanishing fibre PMD. In contrast, NLPR can appear in fibres with zero PMD. We have previously analysed NLDP by solving the CNLS in the presence of fibre PMD for multi span systems[2]. Here we follow a similar concept while streamline the theory for a single span propagation which reveals the somewhat counter-intuitive NLDP growth with increasing PMD.

CNLS for Anti-symmetric Perturbations

For the sake of simplicity, we consider a cw field $a_{x(y)}$ (probe) residing in a narrow spectral gap of a surrounding, fully un-polarized, and co-propagating strong 'loading' ASE field $A_{x(y)}$ with a boxcar-shaped spectrum (Fig.1). The known CNLS[10] for propagation in z direction can approximate this scenario well:

$$\frac{\partial A^{\Sigma}_{x(y)}}{\partial z} + \beta_{1x}\frac{\partial A^{\Sigma}_{x(y)}}{\partial t} + \frac{j\beta_{2x(y)}}{2}\frac{\partial^2 A^{\Sigma}_{x(y)}}{\partial t^2} + \frac{\alpha}{2}A^{\Sigma}_{x(y)} = j\gamma\left(\left|A^{\Sigma}_{x(y)}\right|^2 + \frac{2}{3}\left|A^{\Sigma}_{y(x)}\right|^2\right)A^{\Sigma}_{x(y)} + j\gamma\frac{1}{3}A^{\Sigma*}_{x(y)}A^{\Sigma 2}_{y(x)}e^{-(+)j2\Delta\beta z} \quad (1)$$

$$with \quad A^{\Sigma}_{x(y)} = A_{x(y)} + a_{x(y)};\ \left|a_{x(y)}\right| \ll \left|A_{x(y)}\right|, \quad (2)$$

where the wavelength-independent α, $\beta_{1x(y)}^{-1}$, $\beta_{2x(y)}$, and γ denote the attenuation, the polarization-dependent group velocities and their dispersion coefficients, and the Kerr nonlinearity of a waveplate, respectively. The high modal birefringence $\Delta\beta$ of regular SSMF induces fast oscillating but ineffective NL interference, known from discussions about the Manakov-PMD equation[11], however negligible in our analysis and are thus left out. The remaining stochastic perturbation is decomposed into a symmetric and an anti-symmetric term (Eq. (3)) with respect to the loading's field. While both describe weak and independently treatable NL interactions in a 1st o. perturbation calculus, the polarization-dependent sign

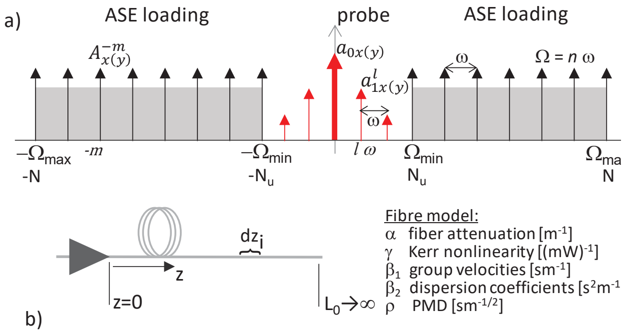

Fig.1: a) Spectral grid for boxcar-shaped ASE spectrum with a probe residing in its centre gap. b) NL single span propagation for NLDP study.

of the anti-symmetric perturbation (right side of Eq. (3)) root-causes opposite phase noises in both principal axes that manifests experimentally as NLDP

$$\frac{\partial a_{x(y)}}{\partial z} + \beta_1\frac{\partial a_{x(y)}}{\partial t} + \frac{j\beta_2}{2}\frac{\partial^2 a_{x(y)}}{\partial t^2} + \frac{\alpha}{2}a_{x(y)} = j\gamma\frac{2}{3}\left(|A_{x(y)}|^2 - |A_{y(x)}|^2\right)a_{x(y)}. \quad (3)$$

Negligibly contributing terms (in $a^2_{x(y)}$, $a_{x(y)}^*$, $A_{x(y)}$, $A_{y(x)}^*$, etc.) of the perturbation are unlisted. The impact of birefringence is disregarded by replacing $\beta_{1x(y)}$ with a polarization-independent group velocity β_1^{-1} but revisited at a later stage.

NL Phase Noises in 1st order Approximation

The right side of Eq. (3) weakly perturbs the probe expressed by adding a 1st o. correction term $a_{1x(y)}(z,t)$ to it. We write the perturbation by means of the undistorted fields of the probe $a_{0x(y)}(z)$ and the loading $A_{0x(y)}(z,t)$. The latter's components with amplitudes $A^m_{x(y)}$ form a comb (Eq. (4)) on an evenly spaced grid with an infinitesimal small angular frequency pitch ω (Fig.1). The probe $a_{x(y)}(z,t)$ in 1st o. development and the ASE field read

$$A_{0x(y)}(z,t) = \sum_{m=-N}^{N} A^m_{x(y)}\, e^{j(k_m z - m\omega t)} e^{-\frac{\alpha}{2}z}; \quad (4)$$

$$a_{x(y)}(z,t) \approx (a_{0x(y)} + a_{1x(y)}(z,t))e^{-\frac{\alpha}{2}z}$$
$$= (a_{0x(y)} + \sum_l a^l_{1x(y)}(z)e^{-jl\omega t})e^{-\frac{\alpha}{2}z} \quad (5)$$
$$with\ A^m_{x(y)} = 0\ \ for\ |m|^2 < N_u, |m|^2 > N; z \geq 0,$$

where $k_m = \beta_1 m\omega + \frac{\beta_2}{2}(m\omega)^2$ stands for the propagation constant of a component at '$m\omega$'. We synthesize the Kerr nonlinearity in Eq. (3) as a sum to address the impact of low frequency beat noises among its terms. Due to phase matching conditions, this noise alone can efficiently interact with the probe and is used to redefine the perturbation term in Eq. (3) as

$$\left|A_{x(y)}\right|^2 - \left|A_{y(x)}\right|^2 \stackrel{\text{def}}{=} \sum_{m,l} \mathcal{A}^{l,m}_{x(y)} e^{-\alpha z} e^{j((\beta_1 l\omega + \beta_2 lm\omega^2)z - l\omega t)} \quad (6)$$

$$with\ \mathcal{A}^{l,m}_{x(y)} = A^{m+\frac{l}{2}}_{x(y)} A^{m-\frac{l}{2}\,*}_{x(y)} - A^{m+\frac{l}{2}}_{y(x)} A^{m-\frac{l}{2}\,*}_{y(x)},\ \begin{matrix} N \geq |m| \geq N_u \\ |l| \ll N_u \end{matrix}. \quad (7)$$

Here '$m\omega$' stands for the frequency spacing between the probe and the two beating ASE tones at $\omega\left(m \pm \frac{l}{2}\right)$, typically in the THz range. Due to coupling inefficiency, any beating at '$l\omega$' among the two noise tones can be neglected when it resides beyond a few tens of MHz. Eq. (3) holds separately for every 'l' and 'm' and its solution can be written by means of a Green's function

$$a^{l,m}_{1x(y)}(z,t) = j\frac{2}{3}\gamma \int_0^z e^{jk_l(z-z_i)-\alpha z_i}\, \mathcal{A}^{l,m}_{x(y)} e^{j((\beta_1 l\omega + \beta_2 lm\omega^2)z_i - l\omega t)} a_{0x(y)} dz_i \quad (8)$$

According to this Ansatz, the NL distortions are generated in fibre sections of incremental lengths, propagate thereafter linearly through the system, and coherently superimpose in the receiver plane. We convert the integral of Eq. (8) into a sum of infinitesimal short waveplates dz_i (Eq. (9)) to analyse NL inter-actions in the presents of PMD. At the span output at L_0 holds

$$\delta^{l,m}_{1x(y)}(L_0,t) = \frac{a^{l,m}_{1x(y)}(L_0,t)}{a_{0x(y)}} = j\frac{2}{3}\gamma e^{j(\beta_1 l\omega L_0 - l\omega t)} \times$$
$$\sum_{\{dz_i\}} \mathcal{A}^{m,l}_{x(y)} e^{-\alpha z_i} e^{j\beta_2 lm\omega^2 z_i} e^{j\frac{1}{2}\beta_2 l^2\omega^2(L_0 - z_i)} dz_i. \quad (9)$$

Alternating the sign of 'l' conjugates its right side, except for its last and typically negligible small exponent $\left(a^{l,m}_{1x(y)}(L_0,t) \approx a^{-l,m}_{1x(y)}(L_0,t)^*\right)$. Hence, pairing contributions at $\pm l$ results into a correction with a 90° phase offset relative to the undistorted probe. Therefore, all pairs of NL distortions stemming from single waveplates, generate pure phase oscillations in the receiver plane $\varepsilon(L_0,t) = \frac{1}{2}(\delta^{l,m}_{1x(y)}(L_0,t) + \delta^{-l,m}_{1x(y)}(L_0,t))$ at a frequency $l\omega$. As $\mathcal{A}^{m,l}_{x(y)} = -\mathcal{A}^{m,l}_{y(x)}$ holds, the oscillations in both orthogonal polarizations are 180° out of phase and cause SOP fluctuations. We define for later purposes a temporal autocorrelation as

$$\varphi^{l,m}_{i,k}(\tau) = \langle \delta^{l,m}_{1x,i}(L_0, t+\tau)\delta^{l,m}_{1x,k}(L_0,t)^* \rangle$$
$$+ \quad \langle \delta^{l,m}_{1y,i}(L_0, t+\tau)\delta^{l,m}_{1y,k}(L_0,t)^* \rangle \quad (10)$$

where the indices i,k denote different waveplates and $\langle\ \rangle$ denotes the averaging over time and fields which involves re-establishing the birefringent fibre features in our model as detailed below.

Phase Noise Correlation in Birefringent Fibre

We model a birefringent fibre as a concatenation of waveplates to determine the correlation among incremental distortions stemming from two different plates and given by Eq. (9). Our x(y)-coordinate system aligns with the fast (slow) axis of a waveplate i.e. it rotates and follows the plates' orientations along the propagation path. Birefringence, originating from axis-specific group velocities ($\beta_{1x} \neq \beta_{1y}$), is incorporated in our model by means of a Jones matrix that transforms the SOPs from the probe and the noise components $A^m_{x(y)}$ when traversing a waveplate. A Jones matrix $\bar{\bar{R}}_i$ of a waveplate shall be given by a unitary matrix

$$\bar{\bar{R}}_i = \begin{bmatrix} R^i_{11} & R^i_{12} \\ -R^{i*}_{12} & R^{i*}_{11} \end{bmatrix}\ with\ \left|R^i_{11}\right|^2 + \left|R^i_{12}\right|^2 = 1. \quad (11)$$

With $a_{0x(y)} := \vec{a_0} = \begin{pmatrix} a_{0x} \\ a_{0y} \end{pmatrix}$ and $\bar{\bar{I}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ a NL distortion generated in waveplate 'k'with length 'dz_k' concisely reads

$$\overrightarrow{a^{m,l}_{1,k}}(L_0,t) = \begin{pmatrix} a^{m,l}_{1x,k}(L_0,t) \\ a^{m,l}_{1y,k}(L_0,t) \end{pmatrix} = j\frac{2}{3}\gamma e^{j(\beta_1 l\omega L_0 - l\omega t)} e^{j\beta_2 lm\omega^2 z_k} dz_k$$
$$\times \left(\prod_{k=i}^{M_{L_0}} \bar{\bar{R}}_i\right) \mathcal{A}^{m,l}_{x,k} \bar{\bar{I}} e^{-\alpha z_k} \left(\prod_{i=1}^{k-1} \bar{\bar{R}}_i\right) \vec{a_0}, \quad (12)$$

where $\mathcal{A}^{m,l}_{x,k}$ und M_{L_0} stand for the undepleted noise components inside of waveplate 'k' constituted by Eq. (7), and the total number of waveplates, respectively. The correlation between contributions from two consecutive wave plates follows as

$$\langle \overrightarrow{a^{m,l}_{1,k+1}}(L_0) | \overrightarrow{a^{m,l}_{1,k}}(L_0) \rangle = \frac{4}{9}\gamma^2 e^{j\beta_2 lm\omega^2(z_k - z_{k+1})} e^{-\alpha(z_k + z_{k+1})} dz_k dz_{k+1} \times$$
$$\langle \mathcal{A}^{m,l}_{x,k+1} \bar{\bar{I}}\bar{\bar{R}}_k\, \overrightarrow{a_{0,k}} | \mathcal{A}^{m,l}_{x,k}\, \bar{\bar{R}}_k \bar{\bar{I}} \overrightarrow{a_{0,k}} \rangle \quad (13)$$

where $\overrightarrow{a_{0,k}}$ and $\langle\ |\ \rangle$ represent the undistorted and undepleted probe field within waveplate 'k' and the field-averaged scalar product, respectively. We restore PMD in our fibre model by using matrices $\bar{\bar{R}}_i, \bar{\bar{R}}_i{}'$ obeying Eq. (11) and transforming wavelength-dependent the probe and ASE fields. After some algebra one finds

$$\langle \mathcal{A}^{m,l}_{x,k+1} \bar{\bar{I}}\bar{\bar{R}}_k\, \overrightarrow{a_{0,k}} | \bar{\bar{R}}_k\, \mathcal{A}^{m,l}_{x,k} \bar{\bar{I}} \overrightarrow{a_{0,k}} \rangle =$$
$$\langle (|R^k_{11}|^2 - |R^k_{12}|^2)\left(\left|R^{k'}_{11}\right|^2 - \left|R^{k'}_{12}\right|^2\right) \rangle\, \mathcal{H}^m \left|\overrightarrow{a_{0,k}}\right|^2, \quad (14)$$

$$\text{with } \mathcal{H}^m = \langle |A^m_x|^2 |A^{m''}_x|^2 + |A^m_y|^2 |A^{m''}_y|^2 \rangle$$

where double primes indicate statistically independent noises. Since $m \gg l$ holds, we can evaluate Eq. (14) just at 'm' but must treat the original noise components at $\left(m \pm {}^{l}/_{2}\right)\omega$ as uncorrelated. The theory of PMD statistics[12] specifies the de-correlation from two SOPs at different wavelengths which provides a correlation between the matrix elements

$$\langle \left(\left|R^k_{11}\right|^2 - \left|R^k_{12}\right|^2\right)\left(\left|R^{k'}_{11}\right|^2 - \left|R^{k'}_{12}\right|^2\right) \rangle = \frac{1}{2} e^{-\frac{1}{3}(\omega m)^2 \tau_\rho^2 \Delta L}, \quad (15)$$

where $m\,\omega$, τ_ρ, and $\Delta L = |z_{k+1} - z_k|$ are the angular frequency spacing between the probe and two noise components, the mean fibre DGD per √length, and the distance between the two waveplates, respectively. PMD effects across frequency intervals of size ~$l\,\omega$ like the spectral width of the distorted probe or spacing's between two beating noise components are negligibly small. Hence, the sum of all incremental phase oscillations correlates as defined by Eq. (10) at the output like

$$\varphi^{l,m}(\tau) = \tfrac{2}{9}\gamma^2 e^{-jl\omega\tau}\mathcal{H}^m \times \sum_{\{dz_i,dz_k\}} e^{j\beta_2 lm\omega^2(z_i-z_k)-\alpha(z_i+z_k)} e^{-\frac{1}{3}(\omega m)^2\tau_\rho^2|z_i-z_k|}\, dz_i dz_k. \quad (17)$$

The above outlined calculus assumes two consecutive birefringent waveplates '$k, k+1$'. But it holds for any pair of further spaced waveplates, indexed 'i, k' with $i \neq k+1$, as well. Since a matrix product $\bar{\bar{R}} = \bar{\bar{R}}_i \dots \bar{\bar{R}}_{k+2}\bar{\bar{R}}_{k+1}$ of intermediately located waveplates can be expressed by a single unitary matrix that fulfils Eq. (11), the conclusion from Eq. (14) will equally hold and leads to Eq. (17).

Fibre PMD Constitutes NLDP

System PMD introduces cut-off conditions via the Gaussian for the number of interacting waveplates addressed by the double sum of Eq. (17). Without this limitation, the sum tends to zero as its complex exponential function causes averaging. For a single span system with $L_0 \gg \alpha^{-1}$ and relatively short wave plates $(\ll \alpha^{-1})$ we replace the double sum with an integral.

Experimentally observed NLDP-caused SOP features such as speed, spectra, and scattering angles are detected after O/E conversion of the optical fields and conveniently reported in Stokes space. To derive such quantities, we limit the optical autocorrelation density by introducing electrical low pass filtering and then convert the result into Stokes space. The density of the optical autocorrelation (Eq. (17)) at $'l\omega'$ in Jones space reads for sufficiently long propagation $(L_0 \to \infty)$

$$\varphi^{l,m}(\tau) \approx \gamma^2\mathcal{H}^m \frac{e^{-jl\omega\tau}}{9\alpha}\int_{-L_0}^{L_0} e^{-\left(\alpha+\frac{1}{3}(\omega m)^2\tau_\rho^2\right)|z|} e^{j\beta_2 lm\omega^2 z}\, dz$$
$$\approx \gamma^2\mathcal{H}^m \frac{e^{-jl\omega\tau}}{9\alpha}\, \frac{\alpha+\frac{1}{3}(\omega m)^2\tau_\rho^2}{(\alpha+\frac{1}{3}(\omega m)^2\tau_\rho^2)^2+(\beta_2 lm\omega^2)^2} \quad (18)$$

and its integration over $'m,l'$ yields the autocorrelation for the total phase noise in the electrical domain. For the sake of simplicity, we assume a fast enough polarimeter with an electrical detection bandwidth $\Omega_e \gg \tau_\rho^2\,\beta_2^{-1}\Omega_{max}$, a sampling rate $\tau_S^{-1} \gg \Omega_e$, and $(\Omega_{min}\tau_\rho)^2 \gg \alpha$, which means a significant PMD impact during the NL propagation as given in long-haul propagation across a multiple span systems. Its 2nd order in τ_S determines the variance of the stochastic SOP speed and reads

$$\varphi_{(0)}^{elec} - \varphi_{(\tau_S)}^{elec} = \int_{-\Omega_{max}}^{\Omega_{max}}\int_{-\Omega_e}^{\Omega_e}(\varphi_{(0)}^{l,m} - \varphi_{(\tau_S)}^{l,m})d\omega l\, d\omega m$$
$$\approx \frac{\gamma^2}{27\alpha\beta_2^2}\,\frac{\Omega_e}{\Omega_{max}-\Omega_{min}}\,\tau_S^2\tau_\rho^2 P_{rep}^2 \quad (19)$$

where P_{rep} stands for the total ASE launch power at the fibre input. To transform the incremental field distortions, determined by Jones calculus, into Stokes space we represent the assumed normalized probe's Jones vector $\overrightarrow{a_{norm}}(L_0,t) = [a_x(L_0,t),\ \ a_y(L_0,t)]^T$ at the fibre output by

$$\overrightarrow{a_{norm}}(L_0,t) = \begin{bmatrix}\cos\vartheta\cos\theta + j\sin\vartheta\sin\theta\\ \sin\vartheta\cos\theta - j\cos\vartheta\sin\theta\end{bmatrix} \quad (20)$$

With $\vartheta = \vartheta_0 + \delta\vartheta(t), |2\vartheta_0| < \pi$ and $\theta = \theta_0 + \delta\theta(t), |4\,\theta_0| < \pi$ where ϑ_0, θ_0 obey known distributions[13] to uniformly cover the Poincare sphere. NLDP causes their small temporary fluctuations $\delta\vartheta(t), \delta\theta(t)$ whose autocorrelations must equal $\langle\delta\vartheta(t+\tau_S)\delta\vartheta(t)\rangle = \langle\delta\theta(t+\tau_S)\delta\theta(t)\rangle = \varphi(\tau_S)$ due to averaging by random birefringence along the propagation path. The corresponding Stokes vector to Eq. (20) $(S_0 = 1, S_1(t), S_2(t), S_3(t))$ can be analysed similarly thus one finds

$$\langle|\vec{S}(t+\tau_S) - \vec{S}(t)|^2\rangle = \tfrac{5}{3}\langle|\overrightarrow{a_{norm}}(t+\tau_S) - \overrightarrow{a_{norm}}(t)|^2\rangle. \quad (21)$$

Identifying $\langle|\overrightarrow{a_{norm}}(t+\tau_S) - \overrightarrow{a_{norm}}(t)|^2\rangle = 2\,(\varphi_{(0)}^{elec} - \varphi_{(\tau_S)}^{elec})$ yields the variance of the NLDP-induced SOPspeed in Stokes space

$$\frac{\langle|\vec{S}(t+\tau_S)-\vec{S}(t)|^2\rangle}{\tau_S^2} \approx \frac{20}{81}\frac{\gamma^2}{\alpha\beta_2{}^2}\,\frac{\Omega_e}{\Omega_{max}-\Omega_{min}}\,\tau_\rho^2 P_{rep}^2. \quad (22)$$

Its PMD dependence differentiates NLDP from NLRP.

Non-Manakovian Propagation Features

We have derived the NLDP-induced SOP speed variance from the CNLS (Eq. (1)) by anti-symmetrizing its perturbation term that weights the impact of the orthogonal polarization by a factor 2/3. A similar set of propagation equations forms the often-used Manakov equation (ME) that utilizes a perturbation with equally rated polarizations as this simplifies algebra and allows for close form solutions in soliton theory. If we would have started our derivation from the ME, not only the magnitude of the NLDP-induced SOP speed but also the ratio between phase noises generated by the symmetric (which we have not discussed here) and the anti-symmetric perturbation would differ. Such shortfall cannot be compensated by an a priori introduction of *'one effective perturbation'* representing both the symmetric and anti-symmetric distortion. Hence, a signal undergoing NLDP does not obey the ME in 1st o. approximation when ordinary and depolarization-caused phase distortions are compared. The ME, highly successfully applied in soliton theory, was originally heuristically found but later 'derived' from the CNLS[7]. Note, this derivation is problematic as it suggests that under the assumption of sufficiently strong and rapidly changing fibre birefringence, i.e. negligible small NL PMD, the CNLS and the ME with a by 8/9 reduced effective nonlinearity yield equivalent solutions. This is not the case for NLDP. The underlying derivation utilizes a locally averaged nonlinearity for the effective NL perturbation. However, in case of NLDP, such a simplification of the CNLS to the ME will not provide consistent results since it does not correctly represent long-range NL interactions, as observed in multiple span systems using Stokes vector spectroscopy[4].

Conclusions

Our outlined analytical model yields under some simplifying assumptions a closed-form solution for NLDP-induced SOP speed in a single span system. An anti-symmetric perturbation operator in the CNLSs generates phase noises that causes SOP fluctuations. A major finding of our model describes the PMD dependence of NLDP which differentiates it from other NL polarization phenomena like NLPR. Our derivations show that in the case of NLDP the CNLSs do not converge towards the Manakow equation as suggested by earlier work.

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