Numerical study on the combination of Probabilistic Shaping and Digital Resolution Enhancer for high baud rate optical communications

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Abstract *We present a numerical study of probabilistic constellation shaping gains in low resolution digital to analogue converters based transceivers, which utilise digital resolution enhancement. A signal to noise ratio gain of 0.75dB for optimized probabilistic shaping factors is achievable.*

Introduction

The past few decades have seen dramatic changes in the architecture of coherent optical communication systems. Supported by digital to analogue converters (DACs) and analogue to digital converters (ADCs), DSP has enabled high baud-rate, quadrature amplitude modulation (QAM) transmission which has multiplied the achievable spectral efficiency over the years. More recently, probabilistic constellation shaping (PCS) has enabled network designers to approach the fundamental Shannon limit [1].

In parallel, the utilisation of low resolution digital to analogue converters (DACs) in high speed optical communication systems has attracted attention in recent years as a way of minimising the operating expenses (OPEx) by minimising power consumption [2]. Reducing DACs resolution increases the impact of quantization noise, which in turn, may limit the systems' performance. However, digital resolution enhancer (DRE) has been introduced in [3] and it demonstrated its ability to transmit and recover up to 64QAM / 44 GBaud using 4-bit DACs with small optical signal to noise ratio (OSNR) penalty compared to a conventional 8-bit DAC.

A recent study of the impact of low resolution DACs and ADCs on probabilistically shaped 64 QAM was previously conducted in [4]. In this paper, we expand this work to analytically investigate the expected improvement of using a DRE with probabilistically shaped 64 QAM at different shaping rates using limited DAC resolutions. We report expected improvements in term of mutual information (MI) for a range of received signal to noise ratio (SNR) from 13 dB to 24 dB for a 4 bit DAC. We also study optimum shaping factors over the aforementioned range of received SNRs.

Pinciple

a) Digital resolution enhancer (DRE)

The DRE concept relies on the fact that quantization noise, which is usually modelled as uniformly distributed white noise [5], can be spectrally shaped to minimise its effect on the signal. Assuming $q(n)$ is quantization error and $h_c(n)$ is the channel impulse response, which is a combination of impulse responses of the optical and electrical transceiver frontend components convolved with the matched filter response, the "effective quantization noise" $q_{eff}(n)$ according to [3] is calculated as

$$\begin{aligned} q_{eff}(n) &= h_c(n) * q(n) \\ &= \sum_{l=0}^{L-1} h_c(n) q(n-l) \end{aligned} \quad \textbf{(1)}$$

where L is the number of channel taps. Due to this correlation, the quantization process can be optimized to inversely match the channel's frequency response by redistributing the frequency components of the quantization noise. Consequently, these high frequency components can be suppressed by the low-pass channel frequency response, and hence, a signal to quantization noise ratio (SQNR) gain can be achieved. The method is based on a pre-quantizer (soft quantizer) and Viterbi algorithm to minimise the mean squared error (MSE), given by

$$MSE = \frac{1}{N} \sum_{n=0}^{N-1} \left| q_{eff}(n) \right|^2 \quad \textbf{(2)}$$

where N is the number of input samples.

b) Probabilistic constellation shaping (PCS)

The principle behind PCS is to optimize the probability of occurrence for all constellation points by giving higher probabilities for inner points and lower probabilities to outer points. It is well known that higher modulation formats require higher signal to noise ratio (SNR) profiles. Conventional square quadrature amplitude modulations (QAMs) support limited optimization options for different SNR values which, significantly, limit the optimization degree of freedom. PCS presents itself in these scenarios as an effective solution to push the performance closer to Shannon's limit, especially when dealing with a Gaussian channel where a SNR gain up to

1.53 dB has been reported [4]. The Maxwell Boltzman (MB) distribution is widely used for numerically studying PCS due to its simplicity and effectiveness especially with AWGN channels. Assuming a M-QAM modulation format, where M is the modulation size, the probability mass functions (PMFs) for symbol (x) generated by a MB distribution follows the equation [6].

$$P_X(x_j) = \frac{e^{-\kappa|x_j|^2}}{\sum_{k=1}^{M} e^{-\kappa|x_k|^2}} ,j = 1,2,..,M \qquad \textbf{(3)}$$

where $\kappa \geq 0$ is the PCS factor. PMFs can be assigned to different symbols by a distribution matcher [1]. Here, we use a constant composition distribution matcher, then invert this process at the receiver side using a reciprocal inverse matcher.

Numerical results and discussion

We simulated an optical back to back (B2B) system following the block diagram shown in Fig.1. The transmitter side begins with QAM generation and a distribution matcher. A 5% pilot overhead is used for synchronisation and carrier recovery aid, and the square root raised cosine (SRRC) pulse shaping factor is 0.1. Subsequently, an optimized pre-emphasis stage is applied to compensate for DAC's frequency response. The channel is approximated to 3 taps (L=3) and 3 soft quantization levels have been assigned to the DRE for quantization noise compensation as explained in [3]. Finally, DACs with a physical number of bits (PNOB) of 4 bits, sampling rate of 92 GSa/s and frequency response set to 23 GHz, 3-dB bandwidth are included.

The receiver side consists of ADCs with PNOB of 8 bits, sampling rate of 92 GSa/s and 23 GHz, 3-dB bandwidth. The receiver's frequency response is compensated by an optimized post-emphasis stage, using a conventional sequence of pilot-based DSP algorithms with a decision directed least mean square algorithm for adaptive filtering and pilot aided phase noise compensation [7]. The DSP chain ends with an evaluation of signal quality metric to calculate the MI. In all simulations, we considered an intradyne dual polarization coherent system. The local oscillator linewidth was set to 100 kHz, and the frequency offset to 100 MHz. Simulations were carried out in two stages: in the first stage, the PCS factor κ was optimized for a 64 QAM, 64 Gbaud signal at different received SNR, to obtain the maximum possible MI. In the second stage, the SNR improvement was evaluated for two of the optimized PCS factors.

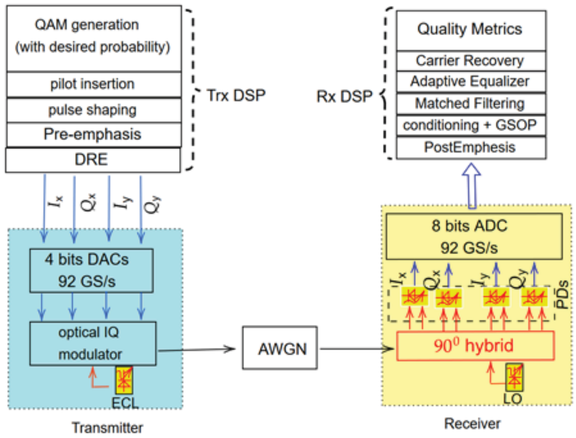

Fig.1 : Block diagram for the optical back to back transmisson system used in our simulations including probabilistic shaping and digital resolution enhancement

a) Probabilistic shaping factor optimization

The received SNR was swept from 13 dB to 25 dB and the PCS factor was tested over a range from 0 (no shaping) to 0.046 (strong shaping). In Fig.2 (a), the MI is normalized for each SNR to the maximum achievable MI at that SNR, $MI_{max_{SNR}}$. The heat map scale bar ranges from zero to unity, corresponding to the lowest and highest achievable MI fo r a given SNR, respectively. It can be clearly seen that higher PCS factors achieve higher MI at low SNR levels and vice versa. Thus, for further investigation, $\kappa = 0.036$ and $\kappa = 0.016$ are chosen to be representative of a strong PCS factor for the SNR range from 17.5-21 dB and a weak PCS factor for SNR range from 13 dB to 15.5 dB, respectively (as indicated in white boxes in Fig. 2(a)).

b) Probabilistic shaping gains for a 4 bits DACs with DRE

Fig.2 (b) shows results of 64 QAM signals at 64 GBaud tested for different scenarios. Applying the DRE with a 4 bit DACs (black star marked curve) shows a SNR gain of 5 dB at MI = 5.5 bits/symbol, compared to the same set-up without using the DRE (yellow diamond marked curve). This gain can be further increased at different SNR levels by using optimized PCS factors. For instance, applying PCS with $\kappa = 0.016$ (blue dash dotted curve) enables up to 0.45 dB of extra SNR. Additionally, a stronger PCS factor such as $\kappa = 0.036$ (purple dashed curve) enables up to 0.75 dB of SNR gain at MI = 4.5 bits/symbol compared to using DRE alone at the same MI level.

c) DRE gains for different probabilistic shaping factors and different DACs resolutions

Fig.2 (c) illustrates expected DRE gains for

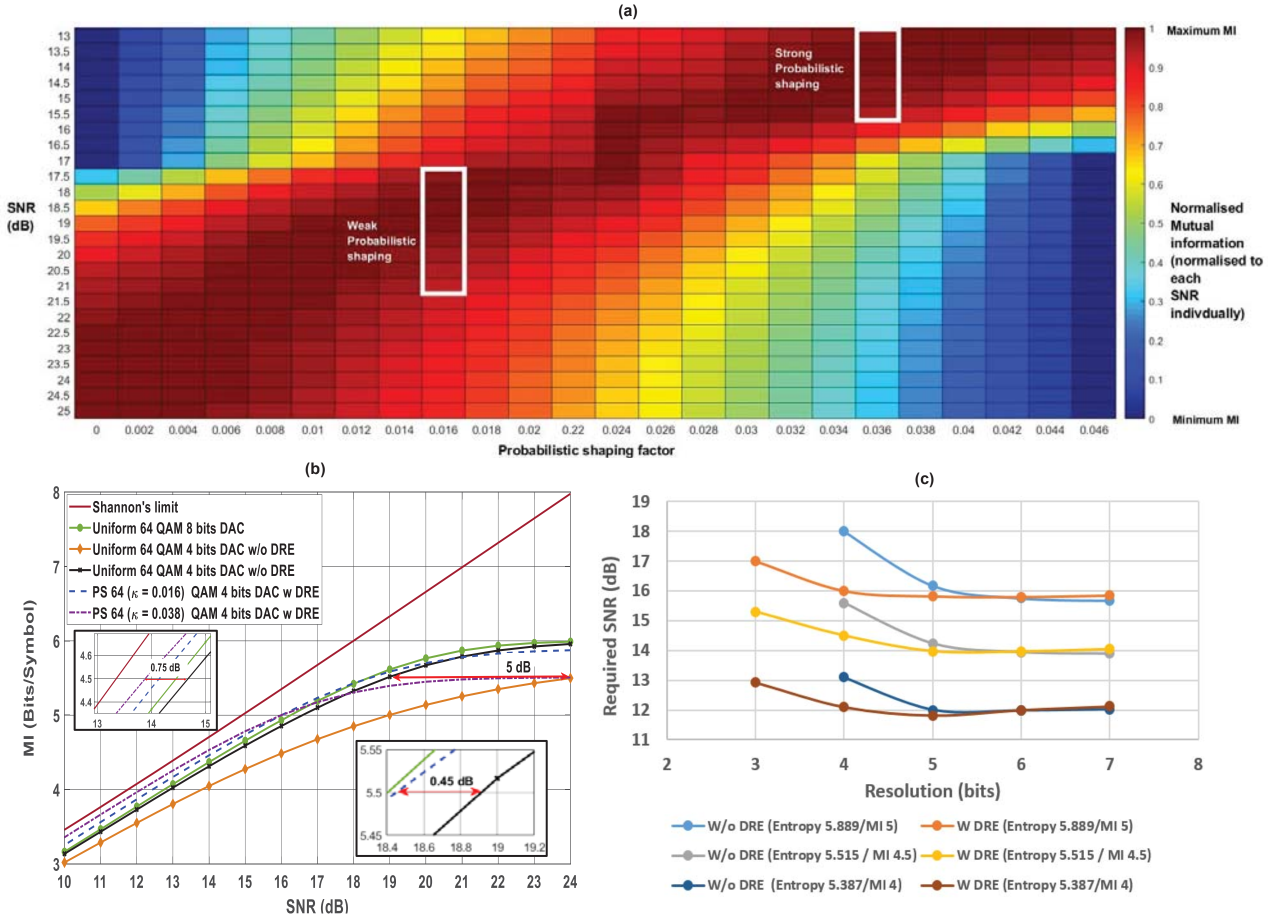

Fig.2 : (a) a heat map shows the performance in terms MI for different SNR vs PCS factors. The scale bar ranges from 0 to 1 where 0 resembles lowest achievable MI and 1 resembles highest achievable MI at each SNR. Optimized strong and weak PCS factors are highlighted **(b)** Performance in term of MI versus SNR for 64 QAM 64 Gbaud in different scenarios compared to Shannon's limit. **(c)** DRE gains for different PCS factors, the comparison is done for three different cases with three different Entropies. A MI reference point which ensures maximum PCS gain has been chosen for each case.

different PCS factors and different DAC resolutions. For a better comparison, we add one extra PCS factor to our set ($\kappa = 0.016\,, 0.036, 0.0435$) with entropies 5.889, 5.515 and 5.387 accordingly. For each entropy (or PCS factor) a MI threshold which ensures maximum PCS gain has been chosen for comparison. The results in Fig.2 (c) show that DRE doesn't enable significant improvement if applied for DACs resolution above 5 bits, however, up to 2 dB of improvement is obtained if applied to 4 bit DACs. Additionally, the DRE shows great improvement if applied to even lower DAC resolution such as 3 bits, however, it is not possible to compare it to a case where no DRE is used since the required SNR would be extremely high to reach the chosen MI threshold. Finally, it can be noticed that DRE gain increases as the PCS factor decreases. This can relate to the fact that PCS improves resilience against noise and distortions, thus, minimise SNR degradation caused by quantization noise. Hence, the DRE gain is limited in such scenarios.

Conclusions

A numerical study has been conducted to investigate the optimization of PCS factors for optical transceivers to increase the SNR gain of using a DRE with low resolution DACs transceivers. Combining PCS with DRE in a 4 bits DACs based transmitter can achieve up to 0.75 dB of SNR gain for $\kappa = 0.036$ and 0.42 dB of extra SNR gain for $\kappa = 0.016$. Additionally, we investigated the expected DRE gain for different PCS factors by varying DACs resolution. DRE shows significant improvement for less than 5 bits DACs resolution. Furthermore, higher gain is observed for lower PCS strengths, thus, a wider support for PCS is achieved for variable rate transceivers that operate with limited resolution DACs. DRE gain can decreases by increases PCS strength.

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References

[1] T. Fehenberger, A. Alvarado, G. Bocherer, and N. Hanik, “On Probabilistic Shaping of Quadrature Amplitude Modulation for the Nonlinear Fiber Channel,” *J. Light. Technol.*, 2016.

[2] C. G. Gruber, “CAPEX and OPEX in Aggregation and Core Networks,” in *Conference on Optical Fiber Communication, Technical Digest Series*, 2009.

[3] Y. Yoffe, E. Wohlgemuth, and D. Sadot, “Digitally Enhanced DAC: Low-Resolution Digital Pre-Compensation for High Speed Optical Links,” in *2018 Optical Fiber Communications Conference and Exposition (OFC)*, 2018, pp. 1–3.

[4] D. Pilori, G. Bosco, and C. Fludger, “Impact of finite-resolution DAC and ADC on probabilistically-shaped QAM constellations,” in *2017 IEEE Photonics Conference (IPC)*, 2017, pp. 433–434.

[5] B. Widrow, I. Kollár, and M. C. Liu, “Statistical theory of quantization,” *IEEE Trans. Instrum. Meas.*, 1996.

[6] G. Bocherer, F. Steiner, and P. Schulte, “Bandwidth Efficient and Rate-Matched Low-Density Parity-Check Coded Modulation,” *IEEE Trans. Commun.*, 2015.

[7] S. T. Le, T. Kanesan, E. Giacoumidis, N. J. Doran, and A. D. Ellis, “Quasi-pilot aided phase noise estimation for coherent optical OFDM systems,” *IEEE Photonics Technol. Lett.*, 2014.