Few-mode Multi-core Fibres: Weakly-coupling and Randomly-coupling

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Abstract Few-mode multi-core fibre (FM-MCF) design suitable for SDM transmission is described. For weakly-coupled FM-MCF, we explain how to determine fibre parameters and investigate the maximum spatial density in a limited cladding diameter. For randomly-coupled FM-MCF, we discuss how to enhance random mode-mixing between different mode-groups.

Introduction

To overcome the capacity limit of single-core single-mode fibres (SMFs), space division multiplexing (SDM) has attracted considerable attention^[1]. Few-mode fibres (FMFs)^{[2],[3]} as well as multi-core fibres (MCFs)^[4] are good candidate for SDM transmission media. In addition, the mode multiplexing and core multiplexing can be combined in MCFs, which is known as a fewmode multi-core fibre (FM-MCF)^[5], and various large capacity transmission experiments using FM-MCFs have been reported^{[6],[7]}. Such SDM fibres can also be distinguished by whether each spatial channel is weakly-coupled or randomlycoupled/strongly-coupled with other channels. In weakly-coupled MCFs, each core is used as an individual waveguide with sufficiently low interference between neighboring cores. On the other hand, in randomly-coupled MCFs, the random mode mixing is intentionally introduced by decreasing the core-to-core distance, resulting in the reduction of modal dispersion^[8].

In this paper, we focus on the FM-MCFs for long-haul SDM transmission. For weakly-coupled FM-MCFs, we explain how to determine the fibre parameters such as core-to-core distance and the number of cores, and investigate the maximum possible spatial density/channel count in a limited cladding diameter. Moreover, for randomly-coupled FM-MCFs, we discuss how to enhance the random mode mixing between different mode groups to suppress the group delay spread (GDS) for multi-mode transmission.

Design parameters of MCFs

Figure 1(a) shows the design parameters of MCFs. The refractive index profile is determined based on the effective area (A_{eff}), the cutoff wavelength, and the differential mode delay (DMD) in each core. The core-to-core distance is determined based on the allowable crosstalk (XT) level between cores. In weakly-coupled MCFs, the XT should be sufficiently small for long distance transmission. On the other hand, in randomly-coupled MCFs, the core-to-core



index profile with low index trench.

distance has to be carefully determined to enhance random mode mixing. The cladding thickness (CT), which is the distance from the core center of the outmost core and cladding edge, is designed to hold the confinement loss to sufficiently low level such as 10⁻³ dB/km^[9]. The number of cores and core arrangement are related to the spatial density, and the upper limit of the cladding diameter (CD) is determined based on the mechanical reliability^[10] and the productivity of fibres.

In order to define the spatial density of MCFs, the core multiplicity factor (CMF)^[9] is one of the parameters which is given as

$$CMF = \frac{N\sum_{i}^{M} A_{eff,i}}{(\pi CD^2/4)}$$
(1)

where A_{eff,i} is the effective area of the i-th mode, M is the number of spatial mode per core, and N is the number of core. The relative core multiplicity factor (RCMF) is the CMF normalized by that for standard SMF which is calculated when N=M=1, A_{eff} =80 μ m² and CD=125 μ m. By using this RCMF, we can compare the spatial density of FM-MCFs.

Weakly-coupled FM-MCFs

Figure 1(b) shows a graded index profile with low index trench considered here. It is widely used for weakly-coupled FM-MCFs^{[11]-[15]}, since the graded-index core is suitable for reducing the DMD and the low index trench is effective for obtaining a low XT between cores and a low bending loss for propagation modes. Table 1 shows typical structural parameters for 2LPmode (3-mode), 4LP-mode (6-mode), 6LP-mode

Tab. 1: Structural parameters of few-mode cores

	2LP	4LP	6LP	9LP
r ₁ [μm]	8.3	9.6	10.8	11.8
r ₂ [μm]	10.0	11.0	12.1	13.0
W/r ₁	0.77	0.67	0.58	0.51
Δ1 [%]	0.62	0.82	1.00	1.20
Δ2 [%]	-0.7	-0.7	-0.7	-0.7



Fig. 2: Bending loss of few-mode cores as a function of CT at 1565 nm, where the bending radius R=140 mm.

(10-mode), and 9LP-mode (15-mode) cores, where the graded index shape factor α is assumed to be 2.0. Each core profile is designed so as to obtain the similar Aeff as that of the fundamental mode of 80 μ m² at 1550 nm and to hold the maximum DMD value to less than several hundred ps/km in the C-band. The trench width W is determined to keep the cutoff wavelength below 1530 nm. It is to be noted that the relative trench width W/r₁ has to be decreased as increasing the number of modes.

The required CT can be estimated by evaluating the bending loss in the outmost core. Figure 2 shows the calculated bending loss of LP₁₁ mode, LP₀₂ mode, LP₂₁ mode, and LP₀₃ mode in the 2LP-, 4LP-, 6LP-, and 9LP-mode cores, respectively, as a function of CT at 1565 nm, where the bending radius is assumed to be R=140 mm. It can be clearly seen that the CT has to be increased as increasing the number of modes, and the CT is required at least 31.8 μ m, 36.7 μ m, 41.6 μ m, and 47.5 μ m, for 2LP-, 4LP-, 6LP-, and 9LP-mode operation, respectively, to keep the bending loss below 10⁻³ dB/km.

The required core-to-core distance Λ can be estimated by evaluating the XT between neighboring cores. Figure 3 shows the averaged XT between LP₁₁ modes (XT₁₁₋₁₁), LP₀₂ modes (XT₀₂₋₀₂), LP₁₂ modes (XT₁₂₋₁₂), and LP₀₃ modes (XT₀₃₋₀₃), in the 2LP-, 4LP-, 6LP-, and 9LP-mode cores, respectively, as a function of Λ at 1565 nm, calculated by using a coupled power theory^[16]. We can see that larger Λ is necessary as increasing the number of modes for achieving similar XT level. For example, if the allowable XT

Tab. 2: Designed parameters of CT and Λ for FM-MCFs

	2LP	4LP	6LP	9LP
CT [µm]	31.8	36.7	41.6	47.5
Λ [μm]	35.4	40.2	44.0	47.9



Fig. 3: XT between two few-mode cores as a function of core-to-core distance at 1565 nm.



Fig. 4: RCMF as a function of CD in FM-MCFs under the condition of XT_{worst}=-27 dB/100 km.

between two cores is -35 dB/100 km, the required Λ is at least 35.4 μ m, 40.2 μ m, 44.0 μ m, and 47.9 μ m, for 2LP-, 4LP-, 6LP-, and 9LP-mode operation, respectively. This XT of -35 dB/100 km between two cores corresponds to the worst-case crosstalk XT_{worst} of ~-27 dB/100 km considering 6 neighboring cores, which is for realizing the 1000 km transmission of QPSK signals with a power penalty of less than 1 dB^[17].

Table 2 summarizes the designed parameters of CT and Λ for FM-MCFs with 3~15 propagation modes in each core. Based on these, the scalability of RCMF in FM-MCFs as a function of CD can be estimated as shown in Fig. 4, where the fibre parameters are shown in Tables 1 and 2, and hexagonal core arrangement is assumed. It is found that the RCMF over 100 is achievable by 6LP-mode 12-core fibre with channel count of 120^[15] and the RCMF can be increased up to ~140 by using 9LP-mode 12-core fibre with channel count of 180 under a condition of CD<250 µm. In theoretically, further increment of spatial density is possible by increasing the number of modes M/cores N, however, the mode dependent loss degradation at a splicing point will be problematic^[15] due to rotational misalignment.

Randomly-coupled FM-MCFs

In multi-mode transmission, GDS originating from the modal dispersion is one of the major since the magnitude of GDS problems determines the complexity of MIMO receiver (and hence, system reach). The random mode mixing in randomly-coupled MCFs is beneficial for reducing the GDS and the GDS in the strong mode mixing regime is proportional to the square root of the transmission distance^[18]. By using coupled single-mode MCFs, very low GDS of several ps/ $\sqrt{\text{km}}$ has been already reported^[19]. On the other hand, in coupled FM-MCFs, it is not easy to realize strong mode mixing between different mode groups^[20], resulting in large GDS.

To estimate the GDS numerically in FMFs/FM-MCFs, the group-delay operator (GDO)^[21] is usually used and it is given as:

$$GDO(\omega) = jT(\omega)^{-1} \frac{\partial T(\omega)}{\partial \omega} \qquad (2)$$

where ω is the angular frequency, $T(\omega)$ is the transfer matrix, and the GDS is defined by the standard deviation of eigenvalues of $GDO(\omega)$. Figure 5 shows a schematic of a coupled FM-MCF with step-index profile considered here, where the core radius is *a*, the relative refractive index difference between core and cladding is Δ , and the core-to-core distance is Λ . It is assumed that the MCF is bent with a uniform bending radius R and the fibre has somewhat of an intrinsic twist with an averaged twisting rate (twisting angle per unit length) γ . To evaluate $T(\omega)$ numerically, a coupled-wave theory^[22] is used. The fibre with the length of L is divided into sufficiently small segments with the segment length of ΔL . Each segment has randomly generated twisting rate γ_l and the twisting angle $\Delta \theta_l$ relative to the previous segment is given by $\gamma_l \Delta L$. γ_l has Gaussian distribution with the mean value of γ and the standard deviation of σ_{γ} . Here, we set the γ , σ_{γ} , and ΔL to π rad/m, 0.1 π rad/m, and 1 mm, respectively^[23], and the calculation results for GDS are averaged over 30 fibre realizations at the wavelength of 1550 nm.

Figure 6 shows the calculated GDS for a coupled 2LP-mode 4-core fibre as a function of core-to-core distance Λ for several different values of *a*, where Δ =0.30 %, R=50 mm, and L=10 km. It is found that the GDS curve has the minimum value at the specific core-to-core distance for each *a*. This is because, at the minimum GDS point, the averaged DMD for all the super modes along the propagation direction in the bent and twisted MCF becomes minimum. For example, when *a*=6.6 µm and Λ =26.5 µm, both the low averaged DMD and strong mode mixing conditions are satisfied, resulting in the low GDS of 40 ps after 10 km propagation.



Fig. 5: Schematic of a coupled FM-MCF with constant bending radius R and twisting rate γ .



Fig. 6: GDS after 10 km propagation in the coupled 2LPmode 4-core fibre with step-index profile as a function of Λ for the bending radius of R=50 mm, where Δ =0.30 %.



Fig. 7: GDS in coupled 2LP-mode 4-core fibres as a function of propagation distance for *a*=6.4 μ m and 6.6 μ m, where Λ =26.5 μ m, Λ =0.30 %, and R=50 mm.

Figure 7 shows the GDS in the coupled 2LPmode 4-core fibres as a function of propagation distance, where Λ =26.5 µm, Δ =0.30 %, and R=50 mm. The red and blue lines are results for *a*=6.6 µm and 6.4 µm, respectively. We can see that the red line is proportional to square root of propagation distance, on the other hand, the blue line is linearly increased with increasing the propagation distance. From these results, it is clear that the core parameters (*a*, Δ) as well as Λ in coupled FM-MCFs has to be carefully determined to enhance random mode mixing.

Conclusions

FM-MCF design suitable for large capacity SDM has been described. We discussed the scalability of spatial density in weakly-coupled FM-MCFs and presented that the RCMF can be increased up to ~140 by using 9LP-mode 12-core fibre with channel count of 180 with a practical CD. In addition, we presented that it is possible to design randomly-coupled FM-MCFs with low GDS by satisfying the low averaged DMD condition and selecting the appropriate core-to-core distance.

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