Estimating Modal Group Delay Characteristics of Few-Mode Fibres from Fibre Parameters

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Abstract *We propose simple expressions that can estimate the modal group delay characteristics of few-mode fibres from the fibre parameters. Simulations and experiments confirm that our expressions yield characteristics consistent with the theoretical ones.*

Introduction

Mode-division multiplexing (MDM) transmission systems using few-mode fibres (FMFs) are being actively studied for overcoming the capacity limit of current fibre-optic transmission systems based on single-mode fibres[1],[2]. In MDM transmission, the differential modal group delay (DMD) in the FMF is of significant interest, since it increases the complexity of digital signal processing usually applied to recover the transmitted signals[3]. The DMD characteristics can be measured with a time-domain method[4], a frequency-domain method[5]-[7], or a modal interferometric method[3],[8]. While these methods provide accurate DMD values, they require complicated procedures or expensive equipment. From a practical viewpoint, there is a need for an alternative method that is simpler than existing methods.

To develop a simple DMD measurement method, we investigate how to estimate the delay characteristics in FMFs from readily measurable fibre parameters. This work clarifies the relationship between the group delay of each mode and the fibre parameters, and proposes simple expressions for estimating the delay characteristics of the LP_{01} and LP_{11} modes from the parameters. Moreover, we demonstrate through numerical simulations and experiments that the feasibility of the proposed expressions in estimating the modal group delay characteristics.

Model for estimating modal group delay

Modal group delay τ is given by[9]

$$\tau = \frac{1}{c}\frac{d\beta}{dk} \approx \frac{1}{c}\left[N_2 + \left(N_1 - N_2\right)\frac{d(vb)}{dv}\right], \tag{1}$$

where β is the propagation constant of a guided mode, k is the wavenumber, and c is the speed of light in a vacuum. N_1 and N_2 are the group indices of the core and cladding in the fibre, respectively. The term $d(vb)/dv$ is called the normalized group delay. v is the normalized frequency and b is the normalized propagation constant defined as

$$v = ka\sqrt{n_1^2 - n_2^2}, \tag{2}$$

and

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{\beta/k - n_2}{n_1 - n_2}, \tag{3}$$

where a is the core radius of the fibre. n_1 and n_2 represent the refractive indices of the core and cladding, respectively.

Let us consider a circumferentially symmetric optical fibre. We represent the refractive index of the fibre using the following equation

$$n^2(r) = \begin{cases} n_1^2\left[1 - 2\Delta(r)\right] & : \ 0 \le r < a \\ n_1^2\left[1 - 2\Delta_0\right] = n_2^2 & : \ r \ge a \end{cases}, \tag{4}$$

where r is the radial distance. $\Delta(r)$ and Δ_0 are the relative index differences at the radial coordinates of r and 0, respectively. Then, the normalized group delay of an LP_{lm} mode with azimuthal number l and radial number m can expressed as[10]

$$\begin{aligned} \frac{d(vb)}{dv} &= \int_0^a \left(1 - \frac{\Delta(r)}{\Delta_0}\right)\psi_{lm}^2(r) r dr \\ &+ \frac{a^2}{v^2}\int_0^\infty \left(\frac{d\psi_{lm}(r)}{dr}\right)^2 r dr + \frac{a^2}{v^2}\int_0^\infty \frac{l^2}{r^2}\psi_{lm}^2(r) r dr, \end{aligned} \tag{5}$$

where $\psi_{lm}(r)$ stands for the radial electric field distribution of the LP_{lm} mode.

Substituting Eq. (5) into Eq. (1), we obtain an approximate expression of the modal group delay as follows

$$\begin{aligned} \tau \approx \frac{1}{c}\Bigg\{ N_2 + \left(N_1 - N_2\right)\Bigg[&\int_0^a \left(1 - \frac{\Delta(r)}{\Delta_0}\right)\psi_{lm}^2(r) r dr \\ &+ \frac{a^2}{v^2}\int_0^\infty \left(\frac{d\psi_{lm}(r)}{dr}\right)^2 r dr + \frac{a^2}{v^2}\int_0^\infty \frac{l^2}{r^2}\psi_{lm}^2(r) r dr \Bigg]\Bigg\}. \end{aligned} \tag{6}$$

To obtain a simple expression of the modal group delay, we approximate the radial electric field of the LP_{lm} mode by the following higher-order Gaussian function[11]

$$\psi_{lm}(r)=\frac{2}{w_{lm}}\sqrt{\frac{(m-1)!}{(l+m-1)!}}\left(\frac{\sqrt{2}r}{w_{lm}}\right)^{l}$$
$$\times L_{m-1}^{l}\left(\frac{2r^{2}}{w_{lm}^{2}}\right)\exp\left(-\frac{r^{2}}{w_{lm}^{2}}\right), \tag{7}$$

where w_{lm} is the mode field radius of the LP_{lm} mode that corresponds to twice the higher-order Gaussian spot size[12]. $L^{l}_{m-1}(x)$ is the associated Laguerre polynomial and is given by

$$L_{m-1}^{l}(x)=\sum_{i=0}^{m-1}(-1)^{i}\frac{(l+m-1)!}{i!(m-1-i)!(i+l)!}x^{i}. \tag{8}$$

By combining Eqs. (6)-(8), we can obtain approximate expressions of the modal group delay for the fibres with specific refractive index profiles in which our interest is high such as step- or graded-index fibres.

As the simplest example, the approximate expressions, τ_{01} and τ_{11} for the modal group delays of the LP_{01} and LP_{11} modes, in a step-index two-mode fibre can be obtained as follows:

$$\tau_{01}=\frac{1}{c}\left\{N_{2}+(N_{1}-N_{2})\left[1-\exp\left(-\frac{2a^{2}}{w_{01}^{2}}\right)+\frac{\lambda^{2}}{4\pi^{2}n_{1}^{2}\Delta_{0}w_{01}^{2}}\right]\right\}, \tag{9}$$

and

$$\tau_{11}=\frac{1}{c}\left\{N_{2}+(N_{1}-N_{2})\left[1-\exp\left(-\frac{2a^{2}}{w_{11}^{2}}\right)-\frac{2a^{2}}{w_{11}^{2}}\exp\left(-\frac{2a^{2}}{w_{11}^{2}}\right)+\frac{\lambda^{2}}{2\pi^{2}n_{1}^{2}\Delta_{0}w_{11}^{2}}\right]\right\}. \tag{10}$$

For a parabolic-index fibre, the approximate expressions of the modal group delays of the LP_{01} and LP_{11} modes can be obtained as follows:

$$\tau_{01}=\frac{1}{c}\left\{N_{2}+(N_{1}-N_{2})\left[1-\frac{1}{2}\frac{w_{01}^{2}}{a^{2}}+\frac{1}{2}\frac{w_{01}^{2}}{a^{2}}\exp\left(-\frac{2a^{2}}{w_{01}^{2}}\right)+\frac{\lambda^{2}}{4\pi^{2}n_{1}^{2}\Delta_{0}w_{01}^{2}}\right]\right\}, \tag{11}$$

and

$$\tau_{11}=\frac{1}{c}\left(N_{2}+(N_{1}-N_{2})\left\{1+\exp\left(-\frac{2a^{2}}{w_{11}^{2}}\right)-\frac{w_{11}^{2}}{a^{2}}\left[1-\exp\left(-\frac{2a^{2}}{w_{11}^{2}}\right)\right]+\frac{\lambda^{2}}{2\pi^{2}n_{1}^{2}\Delta_{0}w_{11}^{2}}\right\}\right). \tag{12}$$

DMD $\Delta\tau$ between the modes can be obtained by the difference of the modal group delays,

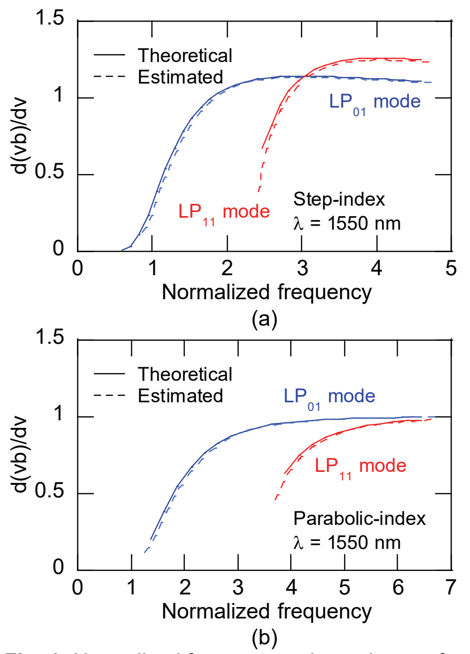

Fig. 1: Normalized frequency v dependence of normalized group delay $d(vb)/dv$ for (a) step-index and (b) parabolic-index fibres.

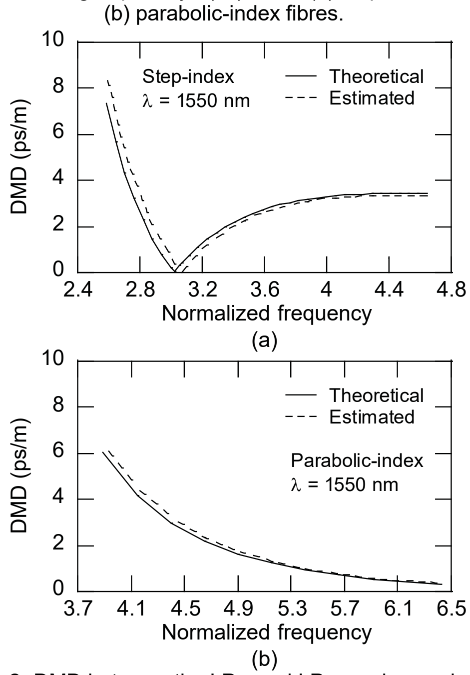

Fig. 2: DMD between the LP_{01} and LP_{11} modes against normalized frequency v for (a) step-index and (b) parabolic-index fibres.

namely $\Delta\tau = |\tau_{01}-\tau_{11}|$. As shown by Eqs. (9)-(12), we have succeeded in clarifying the relationship between the modal group delay of each mode and the structural fibre parameters. Moreover, we found that the modal group delays and the DMD can be expressed in terms of easily measurable fibre parameters: the mode field radius, the core radius, and relative index difference.

Numerical simulations and experiments

To validate the proposed expressions, we simulated the normalized group delay $d(vb)/dv$

and DMD $\Delta\tau$ between the LP_{01} and LP_{11} modes with a help of the finite element method (FEM). Figures 1(a) and 1(b) plot the normalized group delay $d(vb)/dv$ as a function of the normalized frequency v for the step- and parabolic-index fibers, respectively. The blue and red colors plot the results for the LP_{01} and LP_{11} modes, respectively. The solid lines show the theoretical results calculated using the FEM and the index profile. The broken lines show the estimated results from Eqs. (9)-(12) using the fiber parameters. There is good agreement between the estimated results of the normalized group delays as a function of the normalized frequency and the theoretical ones. Figures 2(a) and 2(b) plot the DMD between the LP_{01} and LP_{11} modes as a function of normalized frequency v. The solid and broken lines show the theoretical results of the DMD between the LP_{01} and LP_{11} modes and the estimated results obtained from using Eqs. (9)-(12), respectively. Again, the estimated results almost match the theoretical values.

Next, we prepared two optical fibres, hereinafter referred to as samples A and B. The former had a step-index profile, while the latter had a parabolic-index profile, as shown in Fig. 3. The solid and broken lines plot measured results and the approximated values for numerical simulations, respectively. We measured the DMD using the interferometric technique[3], and compared the measured results with the numerical simulation results. Figures 4(a) and 4(b) show the DMD between the LP_{01} and LP_{11} modes for samples A and B, respectively. The solid lines indicate the theoretical results calculated by differentiating the propagation constants obtained using the FEM, and the broken lines show the estimated values yielded by our expressions. The open circles show the measured results. While the estimated results are slightly different from the measured and theoretical ones, the overall perspective is that they are consistent.

The results shown in Fig. 4 confirm that our proposed expressions can well estimate the modal group delay characteristics from three fibre parameters. Further improvement in the proposed expressions is required to estimate the DMD at a certain wavelength more accurately.

Conclusions

We proposed simple expressions for estimating the modal group delay characteristics in FMFs from a few fibre parameters. We also demonstrated by numerical simulations and experiments that the delay characteristics can be estimated from the mode field radius, the core radius, and relative index difference, which are easily measurable parameters. The expressions proposed herein can be extended to estimate the delay characteristics of any order mode higher than the LP_{11} mode. Our expressions will be useful in characterizing the modal group delay in FMFs.

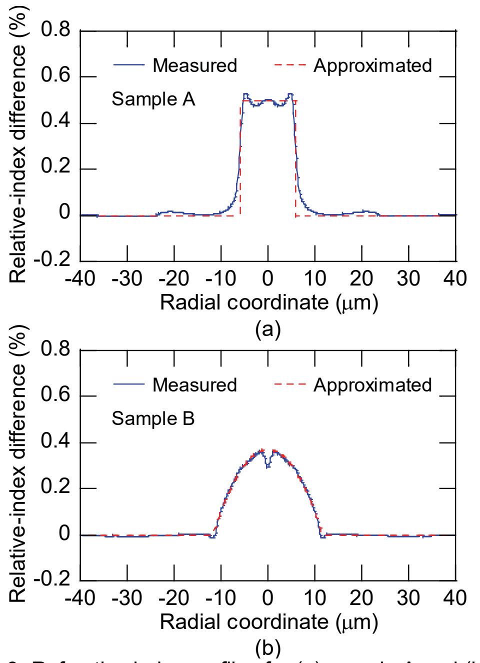

Fig. 3: Refractive index profiles for (a) sample A and (b) sample B.

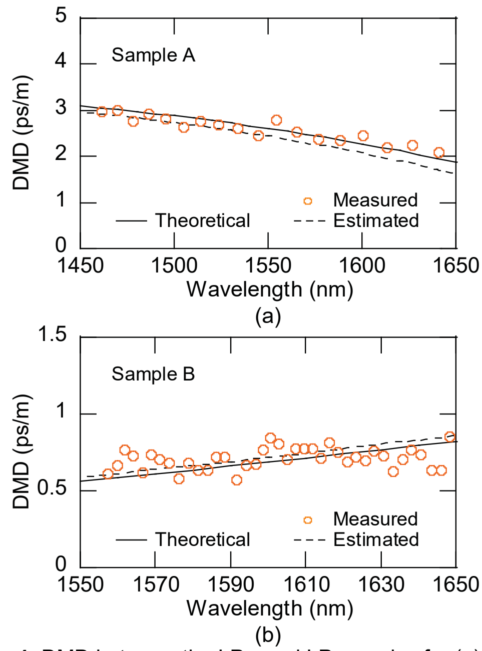

Fig. 4: DMD between the LP_{01} and LP_{11} modes for (a) sample A and (b) sample B.

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