Phase-Retrieving Coherent Reception and its Sample Complexity

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Abstract *Low-complexity coherent receivers based on direct detection are revisited from the viewpoint of sample complexity, number of magnitude samples per symbol in phase retrieval (PR). Recent experimental results on the carrier-less PR receiver by using a 2-D photodetector array are also reported.*

Introduction

Low-complexity coherent receivers based on direct detection, such as the Kramers-Kronig (KK) receiver[1], are a key to migrating coherent optics from backbone networks to edge and access networks. As they rely on direct detection, optical phase information needs to be recovered computationally based on magnitude samples. In other words, they are the solvers for the *phase-retrieval* (PR) problems[2]. From this aspect, it is interesting to revisit such computational coherent receivers according to the sample complexity (SC), i.e., the number of the magnitude measurements required for recovering an unknown phase exactly. For instance, the standard homodyne coherent receiver employ 2-balanced photodetectors (PDs) per channel, thus its SC is said to be 4.

PR Receivers and their SCs

A prominent example of the PR receivers in SC =1 is the KK receiver. When the signal is minimum (or maximum) phase, the log magnitude and phase are related through the Hilbert transform. With the aid of a reference CW signal, the KK receiver fulfils the condition in a practical manner and retrieves the phase deterministically (Fig. 1a). Actually, the minimum phase is necessary for the exact PR of arbitrary band-limited waveforms in SC = 1[3]. Many of the SSBI cancellation receivers[4] can be viewed as iterative approximate PR solvers under the condition. There is another condition for unique PR without the minimum phase, however, it imposes certain restrictions on the input in the z-transform domain[5]. It is evident that, for SC = 1, exact PR is impossible without restricting the input signal or employing a reference signal. In addition, although the KK receiver is SC = 1 in concept, >2-fold oversampling and up-sampling are necessary to avoid the aliasing effect in the deterministic PR process[3]. The effective SC is said to be 2 to 6 in practice[3,7].

In SC = 2, we can have an extra intensity sample per unknown. In [8,9], Matsumoto proposed a PR receiver by introducing the complementary sampling through a dispersive medium (Fig. 2b). The sough phase and the intensities before and after the dispersion are related by the transport-of-intensity equation (TIE) deterministically. This SC = 2 approach may relax the condition for the unique PR in SC = 1. However, to avoid the noise enhancement in the finite differential method for solving the TIE, the author adopted a CW signal and over/up-sampling eventually likewise in the KK receiver (e.g., [10]). The effective SC was 8 to 16 in the numerical simulations. Recently, Chen et al. revealed that the carrier-less phase retrieval is possible by using the same architecture, but with an iterative PR algorithm[11]. The authors regarded the dispersion as an alternating projection in the Gerchberg-Saxton algorithm[12] and proposed an iterative algorithm for the dispersion-assisted PR receiver. The carrier-less coherent reception of 30-Gbaud dual-polarization (DP) QPSK signals was demonstrated experimentally. In [13], the PR receiver was further applied to the mode division

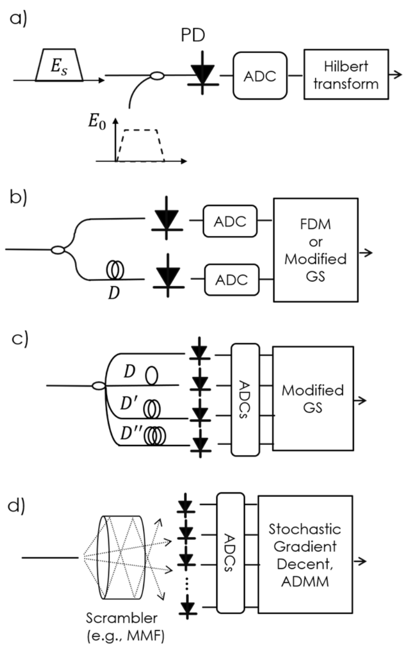

Fig. 1: Computational coherent receivers based on direct detection

multiplexing transmissions. Note that, the effective SC was 4 in the demonstrations by taking the oversampling into account. Interestingly, the PR receiver was extended for SC > 2 by employing multiple dispersive media[14] (Fig.1c). The performance improvement by the parallel projection was demonstrated up-to SC = 4 (i.e., the effective SC=8).

Meanwhile, for SC ≥ 4, there is a completely different theoretical background for unique PR. In the last decade, PR has found interesting theoretical connections with algebraic geometry, low-rank matrix recovery, and compressed sensing[2]. Firstly, by leveraging algebraic geometry, it has been conjectured that $4N - 4$ generic (random) magnitude measurements suffice to retrieve N complex unknowns[15,16]. Thus, for SC≥ 4, the deterministic relationships between phase and intensity, such as KK or TIE, are no more necessary. This drastically improves the design flexibility of the PR receiver. Secondly, the contemporary PR algorithms inspired by the compressed sensing algorithms allow us to solve the general PR problems, namely random quadratic systems of equations, nearly as easy as solving linear systems[17]. The hundreds or thousands of Fourier transformations in the classic PR algorithms are not necessary. In [18], we implemented the random-projection-based PR concept by employing a standard multi-mode fiber (MMF) as an optical scrambler and a two-dimensional PD array (2-D PDA) for high-speed spatial sampling[19] (Fig. 1d). The carrier-less PR reception of 10-Gbaud QPSK, DP-QPSK, 16QAM, and OFDM signals were demonstrated experimentally[18,20] based on a novel low-complexity and robust PR algorithm, which exploits prior knowledge on the modulation format, namely discreteness-aware reweighted amplitude flow (DRAF). The use of the random projection makes the PR receiver scalable and flexible. The PR receiver has potential to be a universal coherent receiver solution for space-division multiplexed (SDM) transmissions as it is insensitive to the SDM schemes; the SDM channels are randomly scrambled anyway.

There are two major questions on the random projection-based PR receiver. 1) How close can it be to the theoretical SC limit in practice. 2) Is the deterministic or non-iterative PR possible? As for the first question, the effective SC was 10 to 12 (i.e., 5 to 6 PDs with 2-fold oversampling per channel) in [20]. Later, we numerically revealed that SC = 4 is possible at least for the QPSK format[21]. However, the PR detection in such low SC regime has not been demonstrated experimentally. In what follows, we will show our recent results on the random projection-based PR reception in the low SC regime. The achievable SC was 7.5 for QPSK signals. There was still a gap from the theoretical limit, but, interestingly, the SC was comparable to the effective SC of some conventional PR receivers. The second question remains open. However, it is worth noting that, recently, some non-iterative PR techniques based on deep neural networks have been proposed and demonstrated for real-time diffraction imaging[22,23]. These approaches are also applicable to the PR receiver based on 2-D PDA in theory.

Experimental Results

Fig. 2 shows the experimental setup for the PR coherent reception of DP-QPSK signals. At the transmitter, a <0.1-kHz-linewidth laser output at 1550.72 nm was modulated by an optical DP-IQ modulator. The modulator was deriven by a 4-channels 10-GSa/s pulse pattern generator (PPG) to generate 10-Gbaud DP-QPSK signals. Note that, the Nyquist filters were not employed. The signal block size was 72 including a cyclic prefix (CP) of 8 symbols long. The CP overhead will be removed by some overlap save methods. The modulated signal was amplified via an Erbium doped fiber amplifier (EDFA) and transmitted over a 1.5-km GI50 MMF (OM3). As in [13,19], the modal coupling and dispersion in the transmission line was exploited as a scrambler. For selective and reproducible mode exitation at the MMF input, a fiber-based mode-multiplexer (Modular Photonics LPMUX3) was employed. The modal crosstalk from the $LP11_{a/b}$

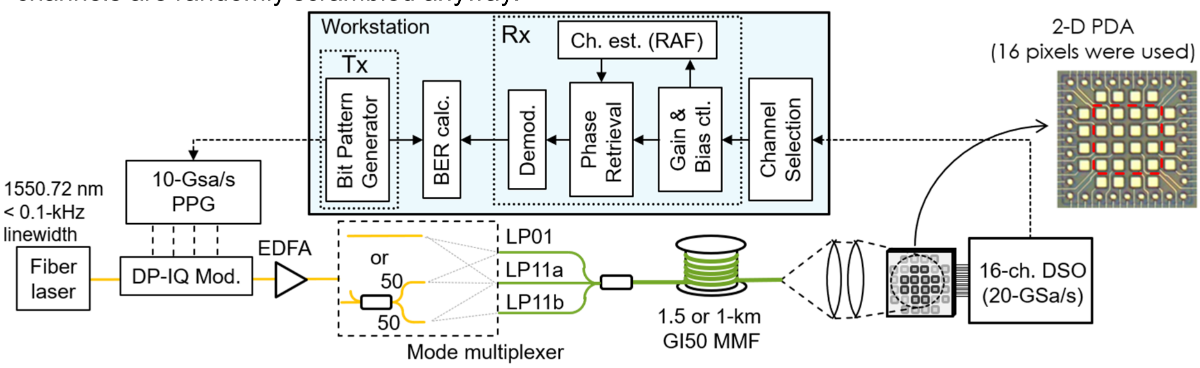

Fig. 2: Experimental Setup for random projection-based PR detection of 10-Gbaud DP-QPSK signals

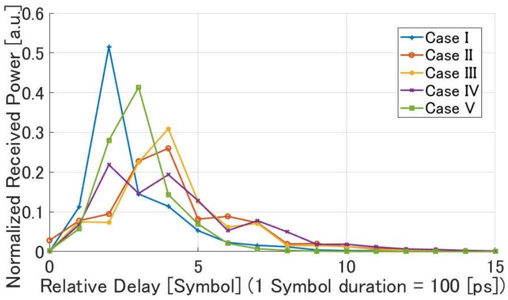

Fig. 3: Power delay profiles

to the LP01 modes in the multiplexier was <-18 dB. We investigated the following five mode-excitation conditions:

- Case I: $LP01$ (SMF) $\rightarrow LP01$ (MMF)
- Case II: $LP01 \rightarrow LP11_a$
- Case III: $LP01 \rightarrow LP11_a, LP11_b$
- Case IV: $LP01 \rightarrow LP01, LP11_a$
- Case V: $LP01 \rightarrow LP11a, LP11b$ (1 km MMF)

For Case III to V, a 50:50 power splitter was implemented in front of the multiplexer. In Case V, a 1-km MMF was employed.

The receiver consisted of a 2-D PDA and collimating lenses. There were no light sources for phase reference. The PDA input power was set to be 15 dBm. (Transimpedance amplifiers were not implemented in the PDA module.) The 2-D PDA consists of 32 PD elements[19]. The pixel size was $30\ \mu\text{m} \times 30\ \mu\text{m}$ with a $44\ \mu\text{m}$ pitch. The 3-dB bandwidth was 11 GHz in average. We used 16 PDs out of 32 as shown in Fig. 2. The PD outputs were oversampled by a 16-ch. digital storage oscilloscope (DSO) at 20 GSa/s. Therefore, 32 intensity samples were received per DP-QPSK symbols, i.e., the effective SC was up-to 16. The intensity samples were porcessed in an offline manner. For PR, PhareADMM (Phare retrieval based on Alternating Direction Method of Multipliers)[24] and DRAF[19,25] algorithms were employed. PhareADMM enables the stable phase recovery in the low SC regime, while DRAF is computationally efficient and outperforms PhareADMM in the high SC regiem[21]. The parameters of the algorithms can be found in [20,21]. The number of iterations was 600 and the weighted maximal correlation method[25] was employed for initialization. Prior to the PR, the channel/scrambler respose was estimated via the pilot-aided PR-based channel esitmation technique[20]. For each mode-excitation condition, we observed the bit-error rate (BER) in 20 different MMF conditions.

Fig. 2 shows power delay profiles (PDPs) of the estimated channel responses. Each PDP is the average of the 32 space-time channels. In Case I, namely central launching, the PDP decays exporentioally. Meanwhile, the modal dispersion and crosstalk in the higher order

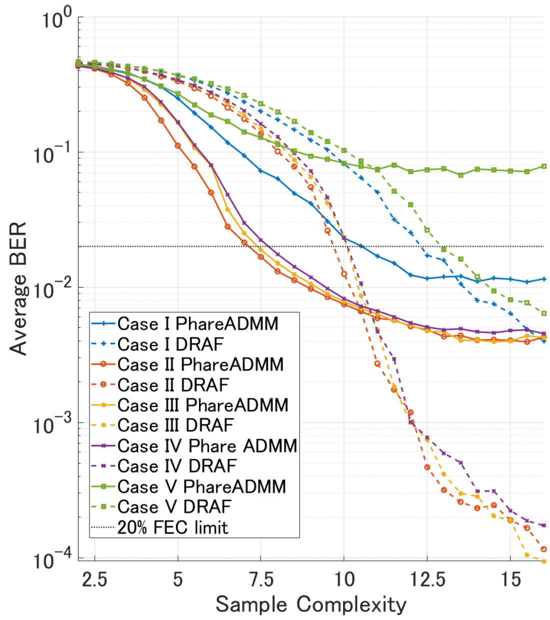

Fig. 4: BER versus sample complexity

mode(s) made the PDP dispersive in Cases II to IV. In Case V, the delay spread was limited due to the shorter MMF length despite of the mode excitation condition.

Fig. 3 shows the average BER performance versus SC for each case. For SC $= N$, the PDs with the N largest received powers were chosen from the 32 PDs. In Cases I and V, the limited channel delay spread resulted in the poor BER. In Case I, the SCs required for achieving $BER < 2.0 \times 10^{-2}$, the threthold for the 20%-overhead forward error correction (FEC), were 11 and 12.5 for PhareADMM and DRAF, respectively. This coincides with our previous results[19]. Meanwhile, in Cases II to IV, the enhanced modal dispersion enbled the stable PR and the BER performance was significantly improved regardless of the modal excitation conditions. The achievable SCs were 7.5 and 10 for PhareADMM and DRAF, respectively. The SC of 7.5 means that 4 PDs per SDM channel with 2-fold oversampling suffice for the PR detection of QPSK signals. Although there is still a notable gap from the theoretical limit, the random-projection-based PR receiver with a simple MMF scrambler could achieve the SC close to some conventional PR receivers.

Conclusions

Computation coherent receivers based on direct detection are revisited from the viewpoint of PR. For SC $\geq$ 4, the carrier-less coherent detection is possible even through random projection in theory. To this end, the random-projection-based PR detection of 10-Gbaud DP-QPSK signals was investigated experimentally in the low SC regime.

Acknowledgements

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References

[1] A. Mecozzi, C. Antonelli, and M. Shtaif, "Kramers–Kronig coherent receiver," Optica, OPTICA 3(11), 1220–1227 (2016).

[2] Y. Shechtman, Y. C. Eldar, O. Cohen, H. N. Chapman, J. Miao, and M. Segev, "Phase Retrieval with Application to Optical Imaging: A contemporary overview," IEEE Signal Process. Mag. 32(3), 87–109 (2015).

[3] A. Mecozzi, C. Antonelli, and M. Shtaif, "Kramers–Kronig receivers," Adv. Opt. Photon., AOP 11(3), 480–517 (2019).

[4] C. Sun, D. Che, H. Ji, and W. Shieh, "Investigation of single- and multi-carrier modulation formats for Kramers-Kronig and SSBI iterative cancellation receivers," Opt. Lett. 44(7), 1785–1788 (2019).

[5] M. Hayes, Jae Lim, and A. Oppenheim, "Signal reconstruction from phase or magnitude," IEEE Trans. Acoust. 28(6), 672–680 (1980).

[6] W. Peng, X. Wu, V. R. Arbab, K. Feng, B. Shamee, L. C. Christen, J. Yang, A. E. Willner, and S. Chi, "Theoretical and Experimental Investigations of Direct-Detected RF-Tone-Assisted Optical OFDM Systems," J. Lightwave Technol. 27(10), 1332–1339 (2009).

[7] T. Bo and H. Kim, "Toward Practical Kramers-Kronig Receiver: Resampling, Performance, and Implementation," J. Lightwave Technol. 37(2), 461–469 (2019).

[8] M. Matsumoto, "A Phase Retrieval Method Using Dispersion for Direct Detection of Biased QAM Signals," in proc. CLEO, paper STu3C.5 (2018).

[9] M. Matsumoto, "Optical Signal Phase Reconstruction Based on Temporal Transport-of-Intensity Equation," J. Lightwave Technol. 38(17), 4722–4729 (2020).

[10] L. Blech, Y. Eldar, C. Antonelli, A. Mecozzi, and M. Shtaif, "The Enhanced Kramers Kronig Receiver," in proc. OFC, paper paper Tu2D.7 (2018).

[11] H. Chen, N. K. Fontaine, J. M. Gene, R. Ryf, D. T. Neilson, and G. Raybon, "Dual Polarization Full-Field Signal Waveform Reconstruction Using Intensity Only Measurements for Coherent Communications," J. Lightwave Technol. 38(9), 2587–2597 (2020).

[12] R. W. Gerchberg and W. O. Saxton, "A practical algorithm for the determination of the phase from image and diffraction plane pictures," Optik 35, 237 (1972)

[13] H. Chen, J. C. Alvarado-Zacarias, H. Huang, N. K. Fontaine, R. Ryf, D. T. Neilson, and R. Amezcua-Correa, "Mode-Multiplexed Full-Field Reconstruction Using Direct and Phase Retrieval Detection," in proc. OFC, paperpaper W4A.5 (2020).

[14] H. Chen, H. Huang, N. K. Fontaine, and R. Ryf, "Phase retrieval with fast convergence employing parallel alternative projections and phase reset for coherent communications," Opt. Lett. 45(5), 1188–1191 (2020).

[15] A. S. Bandeira, J. Cahill, D. G. Mixon, and A. A. Nelson, "Saving phase: Injectivity and stability for phase retrieval," Appl. Comput. Harmon. Anal. 37(1), 106–125 (2014).

[16] A. Conca, D. Edidin, M. Hering, and C. Vinzant, "An algebraic characterization of injectivity in phase retrieval," Appl. Comput. Harmon. Anal. 38(2), 346–356 (2015).

[17] Y. Chen and E. J. Candès, "Solving Random Quadratic Systems of Equations Is Nearly as Easy as Solving Linear Systems," Communications on Pure and Applied Mathematics 70(5), 822–883 (2017).

[18] Y. Yoshida, T. Umezawa, A. Kanno, and N. Yamamoto, "Coherent detection only by 2-D photodetector array: a discreteness-aware phase retrieval approach," in proc. OFC, paper Th4A.3 (2019).

[19] T. Umezawa, T. Sakamoto, A. Kanno, N. Yamamoto, and T. Kawanishi, "High Speed 2-D Photodetector Array for Space and Mode-Division Multiplexing Fiber Communications," J. Lightwave Technol. 36(17), 3684–3692 (2018).

[20] Y. Yoshida, T. Umezawa, A. Kanno, and N. Yamamoto, "A Phase-Retrieving Coherent Receiver Based on Two-Dimensional Photodetector Array," J. Lightwave Technol. 38(1), 90–100 (2020).

[21] Y. Yoshida, T. Umezawa, A. Kanno, K. Inagaki, N. Yamamoto, and T. Kawanishi, "On the Sample Complexity of Phase-Retrieval Receiver Based on 2-D Arrayed Photodetectors," in proc. OFC, paper W4A.2 (2020).

[22] A. Sinha, J. Lee, S. Li, and G. Barbastathis, "Lensless computational imaging through deep learning," Optica, OPTICA 4(9), 1117–1125 (2017).

[23] Y. Nishizaki, R. Horisaki, K. Kitaguchi, M. Saito, and J. Tanida, "Analysis of non-iterative phase retrieval based on machine learning," Opt. Rev. 27(1), 136–141 (2020).

[24] J. Liang, P. Stoica, Y. Jing, and J. Li, "Phase Retrieval via the Alternating Direction Method of Multipliers," IEEE Signal Process. Lett. 25(1), 5–9 (2018).

[25] G. Wang, G. B. Giannakis, Y. Saad, and J. Chen, "Phase Retrieval via Reweighted Amplitude Flow," IEEE Trans. Signal Process. 66(11), 2818–2833 (2018).