Random Polarization-Mode Coupling Explains Inter-Core Crosstalk in Uncoupled Multi-Core Fibers

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Abstract *Existing models for inter-core crosstalk in uncoupled multi-core fibers neglect intra-core random polarization-mode coupling, and conjecture other mechanisms to justify experimental findings. In this work we show that random polarization-mode coupling can explain observed crosstalk values. Good agreement between theory and experiments is obtained.*

Introduction

Since multi-core fibers (MCFs) have proven to be a promising candidate to scale the capacity of future fiber-optic systems, the characterization of inter-core crosstalk became the subject of numerous studies[1]–[12]. However, somehow surprisingly, none of those studies investigated the role of intra-core polarization-mode coupling in the accumulation of the crosstalk along the fiber, but rather ascribed it to fluctuations of the cores' effective refractive index, in combination with fiber bending and twist. In a recent work[13] we showed how to include intra-core birefringence in the modelling of inter-core crosstalk. In this work we make a key step forward by connecting the crosstalk to another easily measurable fiber property, that is the polarization-mode dispersion (PMD) of the individual cores. We show that the two are fundamentally related to each other and the crosstalk between two cores can be reduced by either increasing or reducing their PMD, depending on the magnitude of the corresponding propagation-constant mismatch. We use our model to fit experimental data taken on spooled MCFs and show that polarization-mode coupling can explain by itself the measured crosstalk values. By doing so we do not claim that other, previously studied effects are necessarily negligible, but rather that this problem calls for further research efforts, focusing most preferably on deployed MCFs[14].

Model for inter-core crosstalk

In this section we present a model for the linear crosstalk between the cores of a nominally uncoupled multi-core fiber. Keeping in mind that the goal of this work is to clarify the role of random polarization-mode coupling, we do not include polarization-dependent loss and nonlinear effects in the model, as it would be necessary for a comprehensive simulation of data transmission. Also, in order to keep the illustration simple, we concentrate on the interference between two cores, and we describe the electric fields propagating therein by means of their z-dependent Jones vectors $\vec{E}_n(z)$ and $\vec{E}_m(z)$. The evolution of $\vec{E}_n(z)$ obeys the following equation[13],

$$\frac{\partial \vec{E}_n}{\partial z} = i\beta_{0,n}(z)\vec{E}_n + i\frac{\vec{\beta}_n(z)\cdot\vec{\sigma}}{2}\vec{E}_n + i\kappa_{nm}\vec{E}_m. \quad (1)$$

Here, the z-dependent propagation constant $\beta_{0,n}(z)$ accounts for the effect of fiber bending and twist, and is given by the familiar expression[1]

$$\beta_{0,n}(z) = \bar{\beta}_{0,n}\left\{1 + \frac{D_n(z)}{R_b(z)}\cos[\phi_n(z)]\right\}, \quad (2)$$

where $\bar{\beta}_{0,n}$ is the nominal (z-independent) propagation constant of the n-th core, $R_b(z)$ is the bending radius, $D_n(z)$ is its distance from the center of the fiber cross-section, and $\phi_n(z)$ is the corresponding angle. The term $i\vec{\beta}_n(z)\cdot\vec{\sigma}/2$ is the matrix describing local polarization-mode coupling in the same core, where $\vec{\beta}_n(z)$ is the birefringence vector,[15] and $\vec{\sigma}$ is a vector whose elements are the three Pauli matrices, so that $\vec{\beta}_n\cdot\vec{\sigma} = \beta_{n,1}\sigma_1 + \beta_{n,2}\sigma_2 + \beta_{n,3}\sigma_3$. Finally, the coefficient κ_{nm} quantifies the coupling between $\vec{E}_n$ and $\vec{E}_m$, as prescribed by coupled-mode theory[16]. We note that in general the propagation constant $\beta_{0,n}(z)$ accounts also for the fluctuations of the core effective index, as investigated in previous work[2],[17]. These fluctuations can be readily integrated in the model, however, in order to focus on the effect of the random polarization-mode coupling, in this work we do not account for them.

Within a perturbation approach, the solution to

Eq. (1) is obtained by replacing $\vec{E}_m(z)$ with its expression in the absence of crosstalk, $\vec{E}_m(z) = \mathbf{U}_m(z, z_0)\exp\{i\theta_m(z)\}\vec{E}_m(0)$, where $\mathbf{U}_m(z, z_0)$ is the matrix describing polarization-mode coupling between z_0 and z and $\theta_m(z) = \int_0^z \beta_{0,m}(z')\mathrm{d}z'$. Then the average crosstalk power is obtained by averaging the optical intensity $|\vec{E}_n|^2$ over the effect of the sources of randomness. To this end we first perform an average over the random cores' birefringence and denote the result by $P_{nm}(z) = \langle|\vec{E}_n|^2\rangle$, whereas the randomness of bending and twist is accounted for at a later stage. Assuming that each core's birefringence is an independent random process, the result, whose detailed derivation can be found in[13], is

$$P_{nm}(z) = P_m(0)\kappa_{nm}^2 \int_0^z \mathrm{d}z' \int_0^z \mathrm{d}z'' R_n(z'-z'') \\ R_m(z'-z'')e^{i[\Delta\theta_{nm}(z')-\Delta\theta_{nm}(z'')]}, \quad (3)$$

where the functions R_n and R_m describe polarization decorrelation in the two cores, namely $\langle\mathbf{U}_n(z', z'')\rangle = R_n(z'-z'')\mathbf{I}$, with $\mathbf{I}$ denoting the two-dimensional identity matrix. Note that the process of polarization decorrelation is stationary, which results in the dependence of the average $\langle\mathbf{U}_{n,m}(z', z'')\rangle$ on the difference $z' - z''$ only. In general this is not the case for the phase term, which prevents to further simplify the crosstalk power expression. This limitation can be circumvented by noting that polarization effects are characterized by a much shorter length-scale than macro-bending and twist[17]. This can be done by looking at the increment of the crosstalk power, $\Delta P_{nm}(z) = P_{nm}(z + \Delta z) - P_{nm}(z)$. In this case, if Δz is sufficiently smaller than the length-scale over which bending and twist are constant, in the relevant integration area around the peak of R_n and R_m at $z' \simeq z'' \simeq z$, the phase term can be approximated as $\Delta\theta_{nm}(z') - \Delta\theta_{nm}(z'') \simeq \Delta\beta_{nm}(z)(z' - z'')$, with $\Delta\beta_{nm}(z) = \beta_{0,n}(z) - \beta_{0,m}(z)$. With this simplification, provided that at the same time Δz is sufficiently larger than the polarization coherency length-scale, the following expression for the normalized crosstalk power increment is obtained,

$$\frac{\Delta P_{nm}(z)}{P_m(0)} = \kappa_{nm}^2 \mathcal{R}_{nm}[\Delta\beta_{nm}(z)]\Delta z \quad (4)$$

$$\mathcal{R}_{nm}(\Delta\beta_{nm}) = \int_{-\infty}^{+\infty} \mathrm{d}\zeta e^{i\Delta\beta_{nm}\zeta} R_n(\zeta)R_m(\zeta). \quad (5)$$

The avarage normalized crosstalk is finally obtained by further averaging Eq. (4) over the distribution of bending and twist, namely

$$\Delta\chi = \kappa_{nm}^2 \langle\mathcal{R}_{nm}(\Delta\beta_{nm})\rangle\Delta z. \quad (6)$$

We note that this result, yet formally identical to the one obtained in the framework of the *phase-matching* model[4], underpins a totally different propagation effect – random polarization-mode coupling – whose characterization is well established in the context of single-mode transmission systems but has been only partially explored in the case of multi-core fibers.

Fiber birefringence and PMD

The most realistic model for the random birefringence of single-mode fibers is the one known as the Random-Modulus Model[18]. According to this model the birefringence vector accumulates along the fiber as a two-dimensional Ornstein-Uhlenbeck process, and is characterized by two key parameters. These are the birefringence beat length L_B, which characterizes the strength of the local birefringence trough $\langle|\vec{\beta}|\rangle = 2\pi/L_B$, and the correlation length L_C, defined through the birefringence correlation function $\langle\vec{\beta}(z')\cdot\vec{\beta}(z'')\rangle = (16\pi/L_B^2)\exp(-|z'-z''|/L_C)$[19]. Up to values of L_C of the order of $L_B/10$ the birefringence vector statistics is accurately described in terms of white noise[13], in which case the polarization correlation function is

$$R(\zeta) = e^{-\frac{|\zeta|}{\ell}}, \ell = \frac{L_B^2}{2\pi L_C}. \quad (7)$$

We refer to ℓ as the polarization coherency length, and this is related to the fiber mean differential group delay (DGD) through the simple relation[18],[19]

$$\langle\mathrm{DGD}\rangle = \kappa_{\mathrm{PMD}}\sqrt{z} = \frac{8}{\omega\sqrt{3\pi\ell}}\sqrt{z}, \quad (8)$$

where by ω we denote the carrier frequency and κ_{PMD} is the familiar PMD coefficient characterizing the square-root growth of the mean DGD. Use of (7) into (5) yields[13]

$$\mathcal{R}_{nm}(\Delta\beta_{nm}) = \frac{2\ell_{\mathrm{eq}}}{1 + \Delta\beta_{nm}^2\ell_{\mathrm{eq}}^2}, \quad (9)$$

with $\ell_{\mathrm{eq}} = (\ell_n^{-1} + \ell_m^{-1})^{-1}$, which establishes a simple relation between crosstalk and PMD. In particular, assuming for illustration simplicity the same

	Core 2	Core 3	Core 4	Core 5	Core 6	Core 7
$\Delta n_{\mathrm{eff}}/n_{\mathrm{eff}}$ [%]	0.0085	0.004	0.009	0.001	0.01	0.001
κ_{PMD}[ps/km$^{1/2}$]	0.28	0.2	0.3	0.28	0.28	0.2

Fig. 1: Table showing the values of the core-specific parameters used to fit the data in Fig. 2: the change in effective refractive index relative to the center core (where the effective index of the center core was set to 1.46), and the PMD coefficient. For the center core $\kappa_{\mathrm{PMD}} = 0.28$ ps/km$^{1/2}$.

PMD for both cores, this would be

$$\mathcal{R}_{nm}(\Delta\beta_{nm}) = \frac{2}{\Delta\beta_{nm}} \frac{\frac{32\Delta\beta_{nm}}{3\pi\omega^2\kappa_{\mathrm{PMD}}^2}}{1+\left(\frac{32\Delta\beta_{nm}}{3\pi\omega^2\kappa_{\mathrm{PMD}}^2}\right)^2}, \quad (10)$$

which shows the existence of a critical PMD value for which $\mathcal{R}_{nm}(\Delta\beta_{nm})$ reaches a maximum

$$\overline{\kappa}_{\mathrm{PMD}} = \sqrt{\frac{32\Delta\beta_{nm}}{3\pi\omega^2}} = \sqrt{\frac{32\Delta n_{\mathrm{eff}}}{3\pi c\omega}}. \quad (11)$$

For a relative effective-index difference of the order of 0.001% at $n_{\mathrm{eff}} = 1.46$, the critical PMD value is of about 0.37 ps/km$^{1/2}$ in the C-band. It is interesting to notice that for this value of κ_{PMD}, the polarization coherency length ℓ is of the order of 1 cm. Values of κ_{PMD} in this range (and beyond) – much larger than in typical single-mode fibers that are used today – have been reported in the most recent work on the characterization of PMD in multi-core fibers[20], which indicates that polarization decorrelation in these fibers can occur over a much shorter length-scale than one would expect from the single-mode experience. This point is extremely important in consideration of the fact that scalar (phase-matching-based) models for the crosstalk in multi-core fibers required correlation lengths of the order of 1 cm to justify experimental data[4]. The PMD values measured in[20] suggest that the length-scale of polarization-mode coupling is compatible with this requirement and therefore may play a key-role to explain the spatial dynamics of inter-core crosstalk.

Comparison between theory and data

In this section we use the proposed model to fit experimental crosstalk data reported in[21]. These were taken on a 100-m 7-core fiber, where the center core was excited with CW light and the power at the output of each of the outer cores was measured. This procedure was repeated several times by spooling the fiber about bobbins of increasing radius, so as to test the dependence of the crosstalk on the bending radius. For the purpose of fitting the data, we assumed the same distance $D_n = 39.2\mu$ from the fiber axis for the six

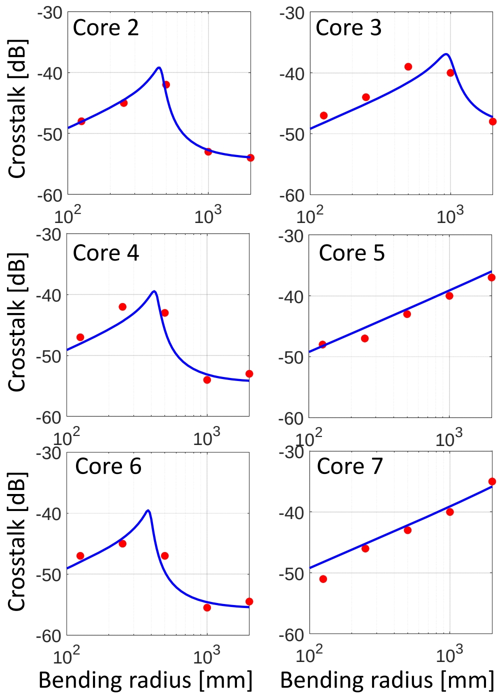

Fig. 2: Average crosstalk versus the bending radius R_b for each of the outer cores of a 7-core fiber. The dots are data points from[21] and solid lines are a plot of Eq. (6).

interfered cores and the same coupling coefficient $\kappa_{nm} = 0.018$ m^{-1}. We only let the PMD coefficient κ_{PMD} and the change in effective refractive index relative to the center core $\Delta n_{\mathrm{eff}}/n_{\mathrm{eff}}$ vaty from core to core. The numerical values used for the fit are shown in the table of Fig. 1. The comparison between theory and experimental data is shown in Fig. 2, where the crosstalk measured in each of the outer cores is plotted versus the deterministic bending radius R_b. The dots reproduce the data points of[21], whereas the solid lines are a plot of Eq. (6), with $\Delta z = 100$ m. The averaging of $\mathcal{R}_{nm}(\Delta\beta_{nm})$ over the effect of the random fiber twist was performed by assuming a uniform distribution between 0 and 2π for the cores' angular position $\phi_n(z)$ in Eq. (2), consistent with previous studies[17]. The good agreement between theory and data is self evident.

Conclusions

We have shown that random polarization-mode coupling provides a self-consistent explanation of experimental crosstalk data measured in nominally uncoupled multi-core fibers. Our results indicate that the inter-core crosstalk is fundamentally entangled with the individual cores' PMD.

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