Low-PAPR Polarization-Time Code with Improved Four-Dimensional Detection for PDL Mitigation

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Abstract *We apply a polarization-time pre-coding together with an improved four-dimensional (4D) detection for polarization dependent loss mitigation. By taking advantage of the correlation between the noises, the proposed scheme outperforms state-of-the-art PDL mitigation methods.*

Introduction

Polarization Dependent Loss (PDL) impairs the performance and reduces the margin of the coherent optical systems. The main source of PDL is in-line components which can cumulatively result in several dB of loss for typical fiber links. In order to increase the transmission capacity, several PDL-mitigating pre-coding techniques have been proposed and investigated [1]-[5] to improve the worst-case performance. On the other hand, the pre-coding increases the signal cardinality observed at the receiver side which in most cases results in high peak-to-average power ratio (PAPR) value. It is well known that signals with high PAPR suffer from nonlinearities of the transmitter devices [6]. A polarization-time (PT) code based on the number theory (NT) with relatively low complexity and low PAPR, herein referred to as NT code, is proposed and investigated in [1]. It is shown that this pre-coder produces smaller PAPR compared to the Golden and Silver codes [7]-[8] and has better nonlinear tolerance. In terms of PDL mitigation its performance is almost similar to the Silver code which is known as the best pre-coder for PDL mitigation in terms of bit error rate (BER) performance.

In this paper, we study the statistical properties of the noise at the receiver side if the NT code is applied at the transmitter side. We prove that after compensating the linear channel effects, and decoding the pre-coder, we come up with a four complex noise elements that they can always be divided into two pairs with non-zero correlation between elements of each pair. Then we use the fact that for a received vector of two complex elements at a given signal-to-noise ratio (SNR), the BER is a decreasing function versus the noise correlation if we use four real-dimension decoders.

Here we propose a method that pairs the received elements with the maximum noise correlation to get the best performance. In the proposed method, two elements with the largest noise correlation from the set of four transmitted elements in the first and the second time slots of X and Y polarizations are determined and then we decode the highly correlated elements jointly using a four dimensional (4D) decoder. Simulation results show that in presence of 6dB PDL, the proposed scheme can improve the Q factor performance of the worst-case scenario compared to the uncoded scheme by approximately 0.8 dB at the target SNR. The improvement due to the proposed 4D decoding compared to using only the pre-coder with 2D decoding is approximately 0.3 dB.

Principle

Let $t_{x,n}/t_{y,n}$ and $t'_{x,n}/t'_{x,n}$ are used to show $\mathrm{n^{th}}$ symbol of X/Y polarization before and after pre-coding $\mathbf{T}$, respectively. We will use the following pre-coder for PDL mitigation [1]:

$$\begin{bmatrix} t'_{x,n} \\ t'_{y,n} \\ t'_{x,n+1} \\ t'_{y,n+1} \end{bmatrix} = \mathbf{T} \begin{bmatrix} t_{x,n} \\ t_{y,n} \\ t_{x,n+1} \\ t_{y,n+1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} t_{x,n} \\ t_{y,n} \\ t_{x,n+1} \\ t_{y,n+1} \end{bmatrix}. \qquad (1)$$

Let assume receiver compensates the effect of the linear channel and carrier phase noise, thus we have

$$\begin{bmatrix} r'_{x,n} \\ r'_{y,n} \\ r'_{x,n+1} \\ r'_{y,n+1} \end{bmatrix} = \begin{bmatrix} t'_{x,n} \\ t'_{y,n} \\ t'_{x,n+1} \\ t'_{y,n+1} \end{bmatrix} + \begin{bmatrix} z'_{x,n} \\ z'_{y,n} \\ z'_{x,n+1} \\ z'_{y,n+1} \end{bmatrix}, \qquad (2)$$

where $\begin{bmatrix} r'_{x,n} & r'_{y,n} & r'_{x,n+1} & r'_{y,n+1} \end{bmatrix}^T$is the equlized received signal we assume the noise vector $\begin{bmatrix} z'_{x,n} & z'_{y,n} & z'_{x,n+1} & z'_{y,n+1} \end{bmatrix}^T$is a complex Gaussian random vector with zero mean and covariance matrix

$$\mathbf{C}' = \begin{bmatrix} \sigma'^2_x & \sigma'_{xy} & 0 & 0 \\ \sigma'_{xy} & \sigma'^2_y & 0 & 0 \\ 0 & 0 & \sigma'^2_x & \sigma'_{xy} \\ 0 & 0 & \sigma'_{xy} & \sigma'^2_y \end{bmatrix}. \qquad (3)$$

By performing the inverse precoding $\mathbf{T}^{-1}$, we come up with covariance matrix

$$\mathbf{C}'_{pre} = \mathbf{T}^{-1}\mathbf{C}'\mathbf{T}^{-H} = \quad (4)$$

$$0.5\begin{bmatrix} \sigma_x'^2+\sigma_y'^2 & 2\sigma'_{xy} & \sigma_x'^2-\sigma_y'^2 & 0 \\ 2\sigma'_{xy} & \sigma_x'^2+\sigma_y'^2 & 0 & \sigma_x'^2-\sigma_y'^2 \\ \sigma_x'^2-\sigma_y'^2 & 0 & \sigma_x'^2+\sigma_y'^2 & -2\sigma'_{xy} \\ 0 & \sigma_x'^2-\sigma_y'^2 & -2\sigma'_{xy} & \sigma_x'^2+\sigma_y'^2 \end{bmatrix}.$$

Theorem 1: In the presence of PDL, the covariance matrix $\mathbf{C}'_{pre}$ has at least two nonzero elements in each column.

Proof: Each column contains one zero and these three elements: $\sigma_x'^2+\sigma_y'^2$, $2\sigma'_{xy}$, and $\sigma_x'^2-\sigma_y'^2$. It is trivial that $\sigma_x'^2+\sigma_y'^2$ cannot be zero. In the following we prove $2\sigma'_{xy}$ and $\sigma_x'^2-\sigma_y'^2$ cannot be zero simultaneously in the presence of PDL. Let us model the PDL channel as

$$\mathbf{P} = \begin{bmatrix} e^{-\alpha} & 0 \\ 0 & e^{\alpha} \end{bmatrix}\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}, \quad (5)$$

in which α and θ determine the amount of PDL and the state of polarization (SOP) rotation, respectively. The impact of the multiple-input multiple-output (MIMO) equalizer can be modeled as multiplying the received signal with the inverse of the PDL matrix (we are assuming zero forcing equalization). Thus, after MIMO equalization, the received signal is changed to

$$\begin{bmatrix} r'_x \\ r'_y \end{bmatrix} = \begin{bmatrix} t'_x \\ t'_y \end{bmatrix} + \begin{bmatrix} z'_x \\ z'_y \end{bmatrix}, \quad (6)$$

where $\begin{bmatrix} z'_x \\ z'_y \end{bmatrix} = \mathbf{P}^{-1}\begin{bmatrix} z_x \\ z_y \end{bmatrix}$ and $\begin{bmatrix} z_x \\ z_y \end{bmatrix}$ is the noise at the input of the MIMO equalizer. Let $\sigma_x^2 = E(|z_x|^2)$ and $\sigma_y^2 = E(|z_y|^2)$, then one can show that

$$\sigma_x'^2 = E(|z'_x|^2) = \mathrm{E}\left(\left|\cos(\theta)\, e^{\alpha} z_x - \sin(\theta)\, e^{-\alpha} z_y\right|^2\right)$$

$$= cos^2(\theta)e^{2\alpha}\sigma_x^2 + sin^2(\theta)e^{-2\alpha}\sigma_y^2, \quad (7)$$

and

$$\sigma_y'^2 = E\left(|z'_y|^2\right) = \mathrm{E}\left(\left|\sin(\theta)\, e^{\alpha} z_x + \cos(\theta)\, e^{-\alpha} z_y\right|^2\right)$$

$$= sin^2(\theta)e^{2\alpha}\sigma_x^2 + cos^2(\theta)e^{-2\alpha}\sigma_y^2. \quad (8)$$

Thus we have

$$\sigma_x'^2 - \sigma_x'^2 = e^{2\alpha}\sigma_x^2\left(cos^2(\theta) - sin^2(\theta)\right) - e^{-2\alpha}\sigma_y^2\left(cos^2(\theta) - sin^2(\theta)\right). \quad (9)$$

In the presence of PDL ($e^{2\alpha}\sigma_x^2 \neq e^{-2\alpha}\sigma_{xy}^2$), the expression (9) can be zero only when θ equals $k\pi + \frac{\pi}{4}$.

Also, we can show that

$$\sigma'_{xy} = \mathrm{E}\left(z'_x z_y'^H\right) = E\left(\left(\cos(\theta)\, e^{\alpha} z_x - sin(\theta)e^{-\alpha} z_y\right)\left(\sin(\theta)\, e^{\alpha} z_x + \cos(\theta) e^{-\alpha} z_y\right)^H\right), \quad (10)$$

thus we can conclude that

$$\sigma'_{xy} = 0.5\sin(2\theta)(e^{2\alpha}\sigma_x^2 - e^{-2\alpha}\sigma_y^2). \quad (11)$$

In presence of PDL, the expression in (11) is zero for $\theta = k\pi/2$.

By using these equations, we can see σ'_{xy} and $\sigma_x'^2 - \sigma_y'^2$ cannot be zero simultaneously.

Theorem 1 expresses that we can always pair the four received signals in two pairs with correlated noise.

Conjecture 1: Consider a multiple-input single-output channel as $\mathbf{y} = \mathbf{x} + \mathbf{z}$ where $\mathbf{x}, \mathbf{y},$ and $\boldsymbol{z}$ are 2×1 vectors. Let elements of $\mathbf{x}$ are generated from a constellation set with unit power and $\mathbf{z} \sim CN(0, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix})$. For given σ_1^2 and σ_2^2 and a maximum likelihood 4D decoder, the BER is a decreasing function of $|\sigma_{12}|$.

Fig. 1 provides an evidence for Conjecture 1. In this figure, we showed the BER as a function of the noise correlation. The SNR of the first and the second elements are set to 18 and 14 dB, respectively. Here we used joint decoding of elements (over both elements of the received vector) to calculate the BER.

Proposed Method

In this section, we describe how the proposed 4D decoding method works. We use Theorem 1 and Conjecture 1 as follows. Theorem 1 shows if we use matrix $\mathbf{T}$ as the pre-coder at the transmitter, for any arbitrary SOP angle, after removing the effect of pre-coder we can pair the four noise elements $(z'_{x,n}, z'_{y,n}, z'_{x,n+1}, z'_{y,n+1})$ in a way that the correlation between the elements in one pair becomes nonzero. Also based on the Conjecture 1, we will choose pairs in the way that have more correlation to provide better performance.

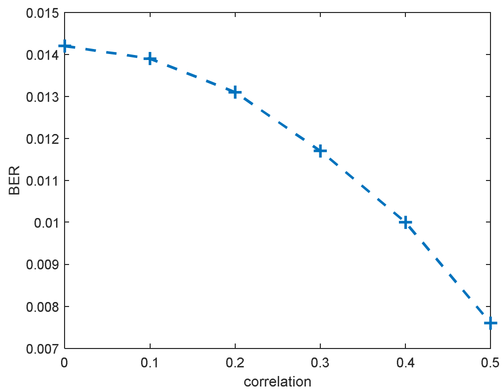
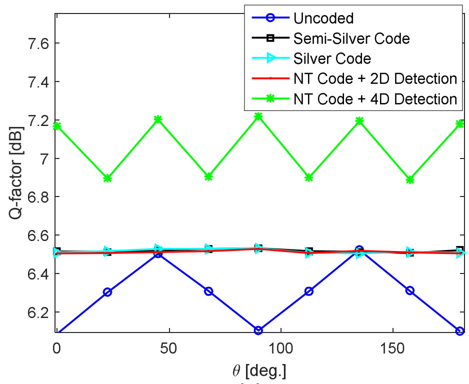


Fig. 1: BER vs. correlation of noise.

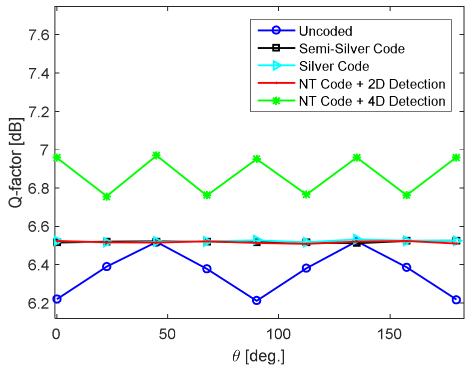
To use the correlation of noise elements, while keeping the complexity low, we need to design a conditional decoder. The following steps describe the proposed 4D decoding at the receiver side when the pre-coder is used at the transmitter side:

a) First, the covariance matrix of noise ($\hat{\mathbf{C}}'_{pre}$) should be estimated. Note that the PDL is a slow impairment thus this covariance matrix can be estimated with high accuracy.

b) In the first column of $\hat{\mathbf{C}}'_{pre}$ we compare between second and third elements.



(a)



(b)

Fig. 2: Q-factor performance vs. SOP angle for **(a)** DP-PCS-16QAM and **(b)** DP-PCS-64QAM.

c) If $\left|\hat{\mathbf{C}}'_{pre}(2)\right| \geq \left|\hat{\mathbf{C}}'_{pre}(3)\right|$: The X and Y polarization in each time slot detected together. In other words, we construct the received vector as $(r_{x,n}, r_{y,n})$ and $(r_{x,n+1}, r_{y,n+1})$ and send them to 4D decoder.

d) If $\left|\hat{\mathbf{C}}'_{pre}(2)\right| < \left|\hat{\mathbf{C}}'_{pre}(3)\right|$: The received vector is paired as $(r_{x,n}, r_{x,n+1})$ and $(r_{y,n}, r_{y,n+1})$ and is sent to 4D decoder.

Results and Discussion

By adding AWGN after 6 dB 1-stage PDL setup, we evaluate the PDL mitigation performance of dual-polarization (DP) probabilistic constellation shaped (PCS)-16QAM (the entropy rate of 3.68 bits/symbol/one-pol), where SNR is set to 13.2 dB, and DP-PCS-64QAM (the entropy rate of 5.5 bits/symbol/one-pol), where SNR is set to 18 dB. Fig. 2 (a) and Fig2 (b) show Q-factor as a function of input SOP angle, θ, for DP-PCS-16QAM and DP-PCS-64QAM, respectively. It was already verified in [1] that the NT code has the lowest PAPR value, and as shown in Fig. 2, when 2D detection is applied, Q-factor is improved and its fluctuation is negligible comparing with the uncoded case. More importantly, it is shown that the NT code with the 4D detection outperforms all other schemes. However, interestingly the fluctuation of Q-factor once more appears after using the 4D detection, and at the angle θ equals integer multiples of 45 degrees, it has the best performance. This fluctuation is due to the correlation of the noise covariance matrix change with the angle θ. We can see employing pre-coder for PDL mitigation provides around 0.5 dB improvement. By adding our proposed 4D decoder we can achieve an additional 0.3 dB gain at the target SNR. Note that as the SNR increases, the gain of 4D decoding method is increased.

Conclusions

Throughout this paper, we presented a novel 4D decoding technique that substantially improves the performance of the PDL mitigation pre-coders. Our method uses the fact that for a given SNR in a vector with two complex elements (four real variables), correlation between elements noises can improve the BER if 4D decoder is employed.

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