Label Extension for 32QAM: The Extra Bit for a Better FEC Performance-Complexity Tradeoff

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Abstract *The FEC performance-complexity tradeoff of 32QAM under label extension to 6 bits is analyzed by simulation and experiment. SNR and complexity reductions are up to 0.3 dB and 30%, respectively, at unaltered baudrate and spectral efficiency.*

Introduction

Power consumption of forward error correction (FEC) is a limiting factor of transponders for high throughput optical transmission. Intermediate size quadrature amplitude modulation (QAM) are constellations with 2^m signal points where m is an odd integer, e.g., $2^3 = 8$QAM, $2^5 = 32$QAM, $2^7 = 128$QAM. The achievable spectral efficiency (SE) of intermediate size QAM can be improved by label extension (LE), which extends the binary label by an extra bit to enable Gray labeling[1]–[3]. The advantages of LE for 8QAM and 32QAM were presented in[4],[5] and[3], respectively. However, the extra bit of LE has two implications for the power consumption of the FEC engine: first, for the same FEC overhead, LE requires a higher baudrate to achieve the same data rate, so the FEC engine needs to run at higher speed, increasing the power consumption. Second, at the same baudrate, the extra bit of LE needs to be coded, which again suggests an increased power consumption of the FEC engine.

In this work, we study the performance-complexity tradeoff of FEC under LE. For a fair comparison, we consider the same 32QAM constellation with LE ('LE32QAM') and without LE ('Cross32QAM'). By adjusting the FEC overhead by shortening, we operate both modes at the same SE, so that using the same baudrate results in the same data rate. In particular, with LE, we use a product code (PC). Without LE, we use the same PC, but shortened. We measure the complexity by the number of component code decodings per second and performance by the required SNR for reliable transmission. By simulation, we find that LE improves the performance-complexity tradeoff. In particular, at the same complexity, LE reduces the required signal-to-noise-ratio (SNR) by 0.3 dB. Even when the complexity of LE is reduced by 33%, the required SNR is still lower than without LE. We verify our findings experimentally by a 96Gbaud DP-32QAM B2B experiment.

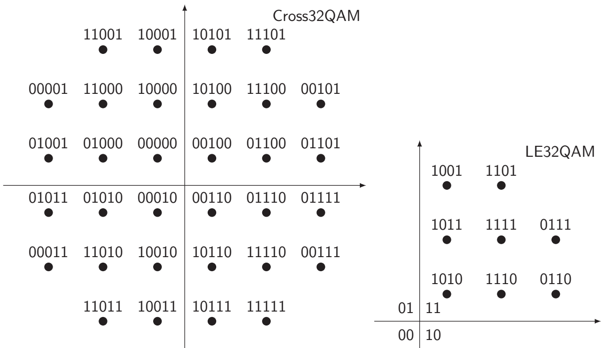

Fig. 1: Cross32QAM (5 bit) and LE32QAM (6 bit). The 6 bit label consists of 2 bits choosing the signs of the I and Q component and 4 bits choosing the signal point within the quadrant. The 4 quadrant bits are output by a 3-to-4 DM, see Tab. 1.

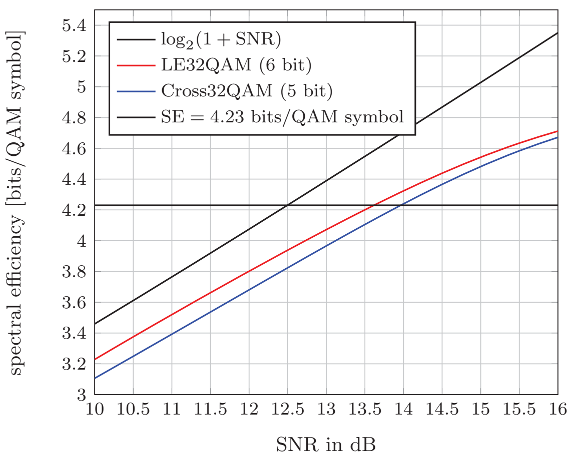

Fig. 2: Achievable rates for Cross32QAM and Le32QAM calculated with (1).

Achievable Rate of 32-QAM Constellations

Consider the 32QAM constellations in Fig. 1. An achievable SE for 32QAM under bitwise demap-

Tab. 1: 3-to-4 DM lookup table[3]

in	out
000	1001
001	1011
010	1010
011	1101
100	1111
101	1110
110	0111
111	0110

ping and binary FEC is[6]

$$\mathsf{SE}^* = \left[\log_2 32 - \sum_{i=1}^{m} \mathbb{H}(B_i|Y)\right]^+, \quad m = 5, 6 \quad (1)$$

where $[\cdot]^+ = \max\{0, \cdot\}$ and where $\mathbb{H}(B_i|Y)$ is the conditional entropy of the ith bit-level and the channel output Y. We display the achievable SE* for the additive white Gaussian noise (AWGN) channel in Fig. 2. We observe that $m = 6$ is around 0.3 dB better than $m = 5$.

Spectral Efficiency and FEC Encoding

Consider now a binary FEC code with rate R_{FEC}. For 32QAM the SE is

$$\mathsf{SE} = 5 - m(1 - R_{\mathsf{FEC}}), \quad m = 5, 6. \quad (2)$$

For Cross32QAM ($m = 5$), any encoding strategy followed by random bit-interleaving can be used. For LE32QAM ($m = 6$), we use the scheme from[3], which is a simple instance of the probabilistic amplitude shaping (PAS) architecture[7]. A 3-to-4 distribution matching (DM) (see Tab. 1) is followed by systematic FEC encoding, so that the DM output bits are preserved at the FEC encoder output and can be used to select the 32QAM signal points within a quadrant, according to the label displayed on the right-hand side in Fig. 1. Additional information bits and the parity bits generated by FEC encoding are used to select the two signs of the I and Q signal point components. A more detailed description is provided in[3]. For interleaving, we use[8], which is compliant with PAS.

FEC Complexity

We consider a product code PC with $(239, 256)$ extended BCH (eBCH) component codes, shortened by s bits. The PC can be represented by a $(256 - s) \times (256 - s)$ square matrix where each row and column is a shortened eBCH codeword. The number of information bits is $(239 - s)^2$ and the length is $(256 - s)^2$, so that the FEC rate is

$$R_{\mathsf{FEC}} = \frac{(239 - s)^2}{(256 - s)^2}. \quad (3)$$

For soft decision (SD) decoding, we use Pyndiah's algorithm[9]. The complexity of decoding an eBCH codeword is determined by the number of parity bits, which is $256 - s - (239 - s) = 17$, independent of the shortening parameter s. In one PC decoding iteration, each row and each column is decoded once, i.e., $2 \cdot (256 - s)$ eBCH decodings are performed. The PC decoding complexity is thus proportional to the number of required eBCH decodings, which is

$$\frac{\text{\#eBCH decodings}}{\text{PC codeword}} = \text{\#SD iterations} \times 2 \times (256 - s). \quad (4)$$

The FEC complexity is proportional to the number of eBCH decodings per second, for which we have

$$\frac{\text{\#eBCH decodings}}{\text{second}} = \frac{\text{\#eBCH decodings}}{\text{PC codeword}} \times \frac{\text{\#PC codewords}}{\text{QAM symbol}} \times \text{\#polarizations} \times \text{baudrate} \quad (5)$$

The number of polarizations and baudrate is constant in our study, so we drop the last two factors. For the second factor in (5), we have

$$\frac{\text{\#PC codewords}}{\text{QAM symbol}} = \frac{\frac{\text{\#bits}}{\text{QAM symbol}}}{\frac{\text{bits}}{\text{PC codeword}}} = \frac{m}{(256 - s)^2}. \quad (6)$$

Finally, using (4) and (6), we quantify the FEC complexity by

$$\begin{aligned}\eta &= \frac{\text{\#eBCH decodings}}{\text{PC codeword}} \times \frac{\text{\#PC codewords}}{\text{QAM symbol}} \\ &= \text{\#SD iterations} \times 2 \times (256 - s) \times \frac{m}{(256 - s)^2} \\ &= i \cdot 2 \cdot \frac{m}{256 - s}. \end{aligned} \quad (7)$$

Simulation Results

According to Tab. 2, Cross32QAM and LE32QAM have exactly the same complexity and SE. In Fig. 3, the bit error rate (BER) curves show that the SNR required for achieving a certain SNR is around 0.3 dB lower for LE32QAM.

In Fig. 4, we vary the complexity by changing

Tab. 2: FEC parameters for simulation of Cross32QAM and Le32QAM

	Cross32QAM	LE32QAM	
m	5	6	
s	44	0	
i	3	3	
R_{FEC}	0.846	0.872	Eq. (3)
SE	4.230	4.230	Eq. (1)
η	0.141	0.141	Eq. (7)

the number of iterations and we display the SNR required for a BER below 1×10^{-4}. We note that the complexity of LE32QAM can be reduced by 33%, while still having a lower required SNR than Cross32QAM.

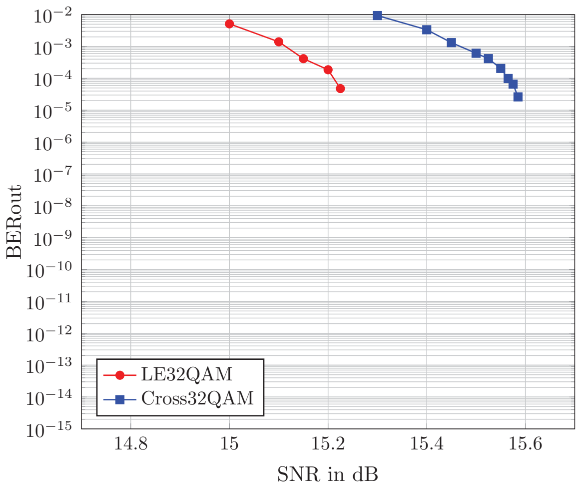

Fig. 3: Bit error rate (BER) curves.

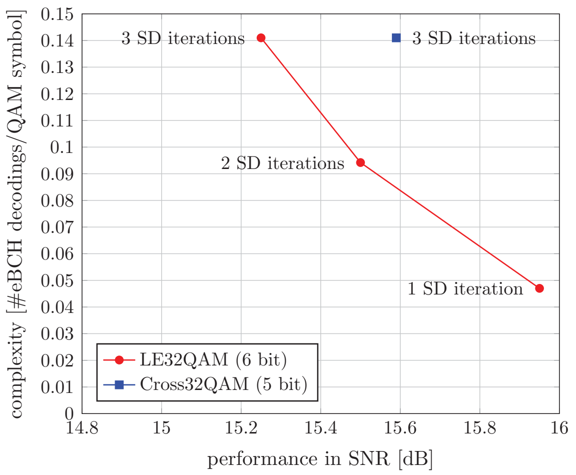

Fig. 4: Performance-complexity tradeoff.

Experimental Results

The experimental verfication is based on a 96Gbaud DP-32QAM signal with gross data rate of 960Gbit/s. Assuming 3.47% for pilot symbols, framing and other training sequences, the net bit rate is 800Gb/s. At the transmitter side, four

Tab. 3: Spectral efficiency and FEC complexity of Cross32QAM and LE32QAM for zero error decoding of B2B experimental measurements at 27.49 dB OSNR

	Cross32QAM	LE32QAM	
m	5	6	
s	65	13	
i	3	2	
R_{FEC}	0.8299	0.8650	Eq. (3)
SE	4.15	4.19	Eq. (1)
η	0.157	0.099	Eq. (6)

BiCMOS 6-bit digital-analog converters (DACs), with 40-GHz 3-dB analog bandwidth are operated at 100-GSa/s and generate a repeated pattern of 76800 samples. Four SHF S804A amplifiers with 60-GHz bandwidth drive the RF signals to a LiNbO3 DP-IQ modulator with 3-dB bandwidth of 32-GHz.

At the receiver side a state-of-the-art optical $90°$-hybrid and four 70-GHz balanced photodiodes (BPDs) are used. The electrical signals are digitized using four 10-bit analog-digital converters (ADCs) operated at 256GSa/s with bandwidth limited to 59-GHz in order to reduce noise. We offline demap the measured noisy 32QAM symbols twice, once with 5 and once with 6 bits, respectively, using the technique suggested in Fig 4 of[10]. For Cross32QAM, we use 3 SD iterations and reduce the FEC rate by shortening until we can decode with no error. For LE32QAM, we use 2 SD iterations and shorten so that we decode with no error. The parameters are displayed in Tab. 3. On the measurement data of the transmission experiment, LE32QAM achieves a higher SE at a lower complexity. This confirms in practice the better performance-complexity tradeoff of LE32QAM as suggested by the simulation results.

Conclusions

We have analyzed the performance-complexity tradeoff of product codes with algebraic component codes for 32QAM with and without label extension (LE). Our findings show that the extra bit of LE actually improves the performance-complexity tradeoff. In particular, in a B2B experiment, LE32QAM achieves for the same measurement a higher spectral efficiency with lower complexity than Cross32QAM without label extension. This suggests that using an extra bit is recommendable for the intermediate size QAM constellations, including 8, 32, and 128 QAM.

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